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REAL TIME ADAPTIVE TRACKING SYSTEM FOR THE COELOSTAT OPTICAL TR--ETC(U)

FEB 77 R V SONALKAR, R L DYGERT, P K SANYAL F30602-75-C-0144

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REAL TIME ADAPTIVE TRACKING SYSTEM FOR THE COELOSTAT OPTICAL TRACKING MOUNT

Pattern Analysis and Recognition Corporation

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AIR FORCE SYSTEMS COMMAND
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REAL TIME ADAPTIVE TRACKING SYSTEM FOR THE COELOSTAT
OPTICAL TRACKING MOUNT

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This report describes the work done at Pattern Analysis & Recognition Corporation under Contract #F30602-75-C-0144 for developing a real time tracking system for the coelostat optical tracking mount at the Verona Test Annex, AOTF (Advanced Optical Test Facility).

The overall system developed, consists of programs for tracking stars and satellites. The star tracking routine requires current values of right ascension and declination of the star to be tracked and may also be used for over

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short time tracking of planets, if so desired. The details of this program were presented in the interim report (PAR Report #75-32) submitted in December 1975.

There are two separate systems for tracking satellites. In the absence of any satellite differential position measuring equipment, Simplified Generalized Perturbation theory is used to predict satellite position. Using NORAD satellite card parameters and a timing input, the program is sequenced to compute the satellite's direction and outputs these angles to the mount servo systems. In the second system a computer-TV interface was fabricated to measure the satellite differential position after obtaining it in the field of view. This measured position is fed back to the computer, which, in turn, uses that as observed error data for a Kalman filter. The filter calculates updated parameters while tracking the satellite position as a function of time.

The report includes a detailed technical description, program documentation, and a system user's manual.

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SECTION 1

INTRODUCTION AND SUMMARY

1.1. INTRODUCTION

This report describes the technical aspects of software developed by Pattern Analysis and Recognition Corporation (PAR) for driving the optical tracking mount, at RADC's AOTF (Advanced Optical Test Facility), Verona test annex with a NOVA 800 computer. The software developed to drive the mount for tracking stars was described in PAR Report No. 75-32 (Contract #F30602-75-C-0144); the hardware designed to interface the computer with the mount was described in another report titled "Coelostat Tracking Mount/NOVA 800 Hardware Digital Interface" (PAR Report No. 75-29). This report describes the technical aspects of the software developed to track satellites of known orbital parameters.

Passive satellite tracking routine makes use of Simplified General Perturbation theory in conjunction with recently computed orbital parameters to predict the position of the satellite at any given instant of time. The section titled "ACQUISITION" describes a spiral search routine designed to perform a systematic search around any calculated satellite position; this will be of great use when inexactness of satellite parameters causes the mount to point away from the true direction and it becomes necessary to acquire the satellite in the field of view. Active satellite tracking describes the software designed not only to predict satellite position, but also to apply suitable corrections to its orbital parameters according to the observed azimuth-elevation angle information of the satellite. The routine uses estimation techniques in the form of a linearized Kalman Filter. The hardware designed to measure the differential azimuth - elevation angles is described in a later section of this report. Program descriptions and a User's Manual are also included.

1.2. THE TRACKING MOUNT

The Coelostat tracking mount permits an observing instrumentation system to remain stationary while the tracker points anywhere within the hemisphere. This particular system allows excursions of approximately ten degrees below the horizon with one meter clear aperture (Figure 1-1).

The mount is a two-mirror system. Both mirrors are mounted on a platform that is rotatable to any azimuth position. One mirror is centered on the azimuth rotational axis and stationary with respect to the platform; it folds the line of sight ninety degrees to become perpendicular to the axis of rotation of the mount. The second mirror, the elevation mirror, is displaced from the azimuth axis and cradled to rotate through the elevation range of approximately 200°. The Coelostat is driven by two independent servos, one for the azimuth axis and the other for elevation (Figure 1-2). By selecting "external

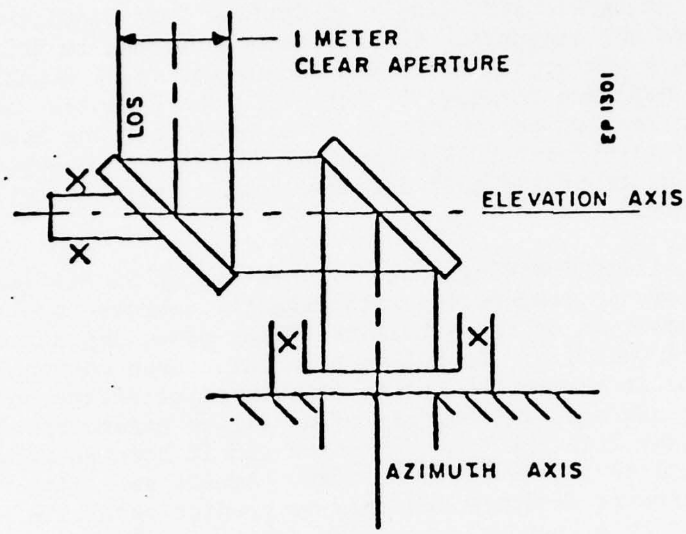


Figure 1-1 Schematic Arrangement of the Coelostat

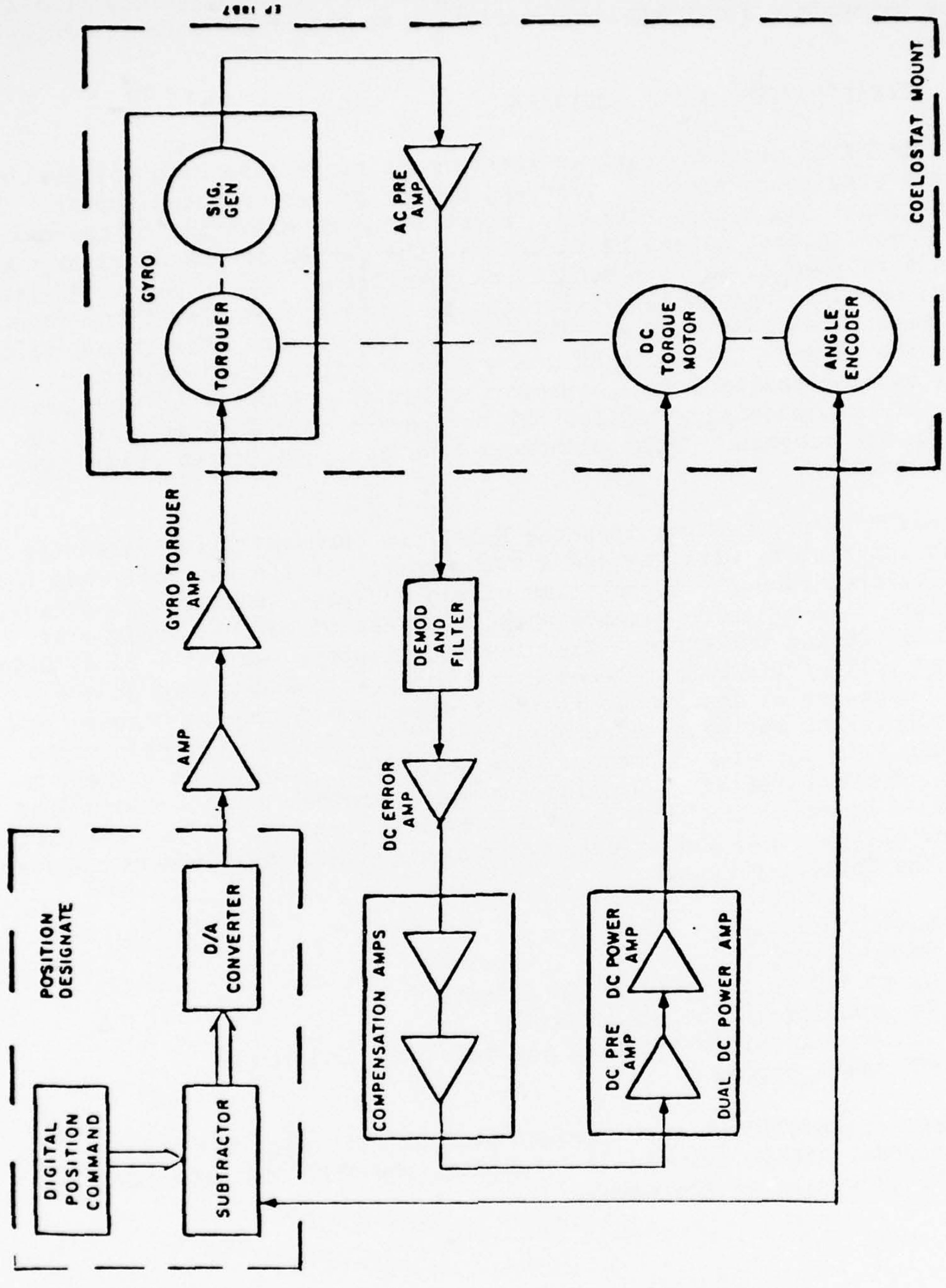


Figure 1-2 Servo-loop

position designate" mode, the mount can be driven by the digital computer. To orient the line of sight in a desired direction, the software supplies the corresponding azimuth and elevation angles to the mount through suitable interface hardware. For detailed description of the mount refer to Reference [1].

1.3. CAPABILITIES OF THE SOFTWARE

The STARTRACK program requires the current right ascension and declination of the star to be tracked, year and day number as operator inputs. The clock time input has been hardwired. Stars above an elevation of approximately 15° will almost always be visible in the center of the field of view of the main telescope, weather conditions permitting. Up to seven-magnitude stars have been photographed. Visibility of stars at low elevations cannot be guaranteed because of haze and atmospheric refraction. The star catalog [2] lists the RA and DEC values of stars brighter than seven-magnitude. Since the catalog tabulates the apparent traces of the stars at each ten-day interval, it is unnecessary to account for precession and nutation of the ecliptic in the program. These phenomena have been considered while tabulating the catalog.

The SATTRACK program requires the NORAD two card satellite parameter set [3] as input along with the day and the year. If the parameter set has been recalculated recently (epoch time within the past week approximately), the satellite will often be visible when it is out of earth's shadow, at least in the viewing telescope. The viewing telescope has its optical axis parallel to that of the main telescope and looks at the sky through the coelostat. Because of its larger field of view (one degree as opposed to five arc-minutes of the main telescope), satellites not in the main scope may be visible in the viewing scope. The satellite can then be brought to the center of the field of view using the Mount Control routines described in the User's Manual. The program is a simplified version [3] of the perturbation theory of Kozai [4] and it does not compute any corrections attributed to any of the following causes.

1. Atmospheric drag
2. Solar pressure
3. Lunar and Solar gravitation effects
4. Ray bending in the atmosphere
5. Structural deformations of the telescope and mount
6. Alignment errors

A number of satellites were tracked successfully with the passive tracking program. While tracking a satellite, the tracking rate never exceeded the slew rate of the mount.

SECTION 2

USER'S MANUAL

2.1. INTRODUCTION

This user's manual is written with the inexperienced user in mind. All commands have been grouped according to their functions and their explanations have been augmented by examples. Unless otherwise specified, all the commands described in this manual perform identical operations in SATTRACK and STARTRACK programs. It is assumed that an experienced mount-operator will be responsible for manning the mount-console.

All commands, except those which are at the RDOS system level (RDOS, SATTRACK, STARTRACK and CARDINPUT), are comprised of four letters. The system will accept just four letters, but the description of commands includes more than just four letters so as to enable the reader to associate the command with the function it performs.

An alphabetical index of commands is included in Section 2.6 for cross-reference.

2.2. PRELIMINARY OPERATIONS

2.2.1. System Initialization

The program is controlled by the Real-time Disk Operating System (RDOS) on the Nova 800 computer system and the following initialization procedure must be adopted.

2.2.1.1.

Insert the disk with the required routine (STARTRACK and SATTRACK) into the disk-drive and turn "on" the power switch key located above the disk-drive.

2.2.1.2.

Press the RUN-LOAD switch to "RUN" and wait till the READY light comes on.

2.2.1.3.

Turn on the power to the CPU,TTY and the line printer, and also the 'SELECT' button on the line printer. The power for the card reader is located on the back of the card reader.

2.2.1.4.

Load the octal number 100033 (1000000000011011 in Binary) in the toggle switch register on the front of the CPU and press program 'LOAD' up.

2.2.1.5.

The TTY should print 'FILENAME?' the proper response to which is RDOS followed by a carriage return (↲).

Operator commands or responses are generally followed by a CR (↲) and system responses have been underlined to avoid confusion.

2.2.1.6.

If the system responds with 'PARTITION OCCUPIED. TYPE C TO CONTINUE', then type only 'C' not followed by carriage return.

2.2.1.7.

Answer the questions regarding date and time by RDOS, which will then respond with 'R' on the TTY. The 'R' indicates that RDOS is in control and READY to accept commands. This completes the sequence of operations for system initialization.

2.2.1.8.

To leave RDOS and shut off the system, type @OUT@ ↲ . The system responds with 'MASTER DEVICE RELEASED.' Then turn off the power on all the devices.

2.2.2. Track Initialization

2.2.2.1. PRINTMANUAL

In case a printout of the tracking manual describing all the commands is required, type PRINTMANUAL ↲. This command prints the complete manual on the line printer.

2.2.2.2. CARDINPUT

To update the satellite list, type CARDINPUT ↲ and answer the questions and read in the cards.

2.2.2.3. STARTRACK

To track stars or any other heavenly body with a slowly varying right ascension and declination, type STARTRACK ↲.

2.2.2.4. SATTRACK

To initiate satellite tracking, type SATTRACK ↲ and then GETP ↲ to get the parameter of the satellite to be tracked (Section 2.3.2.1.). Example:#

---

* When the system prints an asterisk it means that the SATTRACK main command line interpreter (CLI) is in control, as opposed to R which indicates control by RDOS CLI.

R
SATTRACK
INPUT THE DAY AND YEAR (I3, 1X, I2)
325,76
*
GETP
LIST NUMBER
3
*
CONT

2.2.2.4.1. KALCARD

R
KALCARD — This is a command executable at the RDOS level only. This routine allows you to store new cards to the disk in a file. Example:

R
KALCARD
SATELLITES MUST BE DELETED IN THE ORDER IN WHICH THEY APPEAR IN THE LIST.
FIRST SATELLITE TO BE DELETED. — The system asks the operator to delete a satellite if he wishes. If the operator answers with a Ø, no satellites will be deleted.

4 — The operator requests satellite #4 to be deleted.
NEXT SATELLITE? — The system asks for the next satellite to be deleted.
0 — The operator specifies none
INPUT THE NUMBER OF CARDS TO BE READ. — The system asks for a maximum number of cards to be read.
10000 — The operator specifies all of them.

The routine will keep reading cards until 10000 cards have been read or until an end of file card has been reached (not supplied by NORAD). If no end of file card is available, turn off the card reader.

DO YOU WISH THE WORST CASE PARAMETERS TO BE KEPT? — The system asks the operator if he wants the Kalman filter parameters to be the worst case of the past history of the satellite to be the ones kept for tracking. The usual response is 'YES'. After many updates, the answer should be no for one update then yes again.

YES — The operator specifies a yes.
the question
"INPUT THE DIAGONAL TERMS FOR THE PK AND QX MATRIX."

The operator should then give all (10^{-5})'s if he knows nothing about the filter. A more intelligent guess of the parameters would be a satellite of nearly the same card set and use the

diagonal terms for that satellite. These terms are the second set of parameters printed on the line printer for a satellite. The twelve elements could be entered all on one line or on any number of lines. The reason for this question is that this is the first known card set for this satellite, since no past history is known for it. The operator is then given a chance to guess at it.

LOAD $CDR? TYPE ANY KEY WHEN READY — The operator is to load the card reader with the NORAD cards.

↲ — The operator is ready to read in the cards.

MORE CARDS? — The card reader has reached an end of file or has been turned off. The operator can read in more cards by typing "YES".

NO↲ — He has none.

INPUT AN 8 CHARACTER FILE NAME FOR AN INPUT FILE. — The system wants to know where to store the newly created list of parameters. This file must already be present on the disk and must be writable.

KALLIST_ _↲ — The operator specifies the file "KALLIST" as the output file. The trailing spaces are to pad the word. "KALLIST" is the list of parameters used for the Kalman filter.

STOP

R — The operator is back at RDOS.

2.2.2.4.2. KALTRACK

Main Kalman filter tracking routines.

2.2.2.4.3. FILTER

FILTER↲ — This command implements a Kalman filter on the present set of card parameters being used for tracking. The output of this command is a new set of card parameters. The data resulting from the filter is printed on the line printer. The operator is asked the question, "AZ,EL OBSERVED ERRORS?" The operator is being asked for any offsets he wants added on top of the offsets already present in the system. The system then responds with the question: "DO YOU WISH TO UPDATE THE PK MATRIX?" The PK matrix is the system confidence matrix. The normal answer to this question is "NO". An understanding of the filter would be needed to answer yes. Example:

```
*
FILTER↲
AZ,EL OBSERVED ERRORS.(D,M,S)
Ø,Ø,Ø↲
Ø,Ø,Ø↲
DO YOU WISH TO UPDATE THE PK MATRIX?
```

NO↲

* The Kalman filter has been executed, the system time offsets and mount offsets have been used as the error inputs to the filter, and the offsets have been cleared from the system.

2.2.2.5. QTRACK

QTRACK↲ Queue the system to start and stop tracking at operator-specified time instants in universal time (U.T.). Example:

*	Command Line Interpreter (CLI) is in control
QTRACK↲	Operator wishes to queue the system
START TIME, FINISH TIME (UT, H, M, S)	The system asks for start and end times.
15,5,8↲	Operator wants the track to start at 15h. 5m. 8sec. U.T.
15,11,8↲	The track is to stop at 15h, 11m, 8sec, U.T.
AZ, EL START POSITIONS = XXX.XXXXX XX.XXXXXX	Object position at specified start time
AZ, EL FINISH POSITIONS = XXX.XXXXX XX.XXXXXX	Object position at the specified end time
*	Control has been relinquished to CLI

2.2.2.6. QUIT

QUIT↲ This command stops the track and returns control to RDOS. If newly added parameters to the list are to be saved, the list must be written on the disk before issuing this command in STARTRACK or SATTRACK. KALTRACK is always operating from the disk. Example:

*	
QUIT↲	Operator wishes to stop the track
*	The command is executed
STOP	RDOS indicates that the tracking routine is closed
R	RDOS is in control

2.2.2.7. START

START↲ Start the tracking routine again. This has the same effect as coming in from RDOS except that the parameter list contains any additions made since the last disk WRITE↲. All flags have to be reset by the operator. Example:

*	CLI
START↲	
INPUT THE DAY AND YEAR (I3, 1X, I2)	
166,76↲	Day #166 in the year 1976
*	CLI

2.2.2.8. STOP

STOP — Track is stopped but control is retained by the tracking program. This allows the operator to start a new track or have the system waiting for him.

2.2.2.9. CONT

CONT — This command causes the system to continue tracking by computing new AZ,EL angles and transferring them to the mount. This is an active tracking mode and any searches can now be performed.

2.3. DATA STORAGE AND RETRIEVAL

Data required for tracking is stored in a star or satellite list which can be updated as necessary.

2.3.1. Storage

2.3.1.1. XFER

XFER — Transfer the given set of parameters to the appropriate list. The data format is different for a star list and a satellite list. In a star list the parameters are star #, brightness, right ascension (hours, minutes, seconds), declination (degrees, minutes, seconds), so that the units of RA and DEC are the same as those in the star catalog.

The parameters for satellite list are in the same order and units as on the cards ('two card format') and must be entered on two lines. Order of parameters:

line 1: Epoch, year, revolution #, mean motion (n), inclination (i), eccentricity (e)

line 2: Mean anomaly (M), RA of the ascending mode (Ω), argument of perigee (w), first derivative of mean motion ($\dot{n}$), second derivative of mean motion ($\ddot{n}$), satellite #. This command is not valid in KALTRACK.

2.3.1.2. READ

READ — Delete the list in core and replace it with the list on the disk. (Not valid in KALTRACK.)

2.3.1.3. WRITE

WRITE — Write the list in core over the list on the disk. The list in core is left unmodified. No copy of the overwritten list on the disk is retained. (Not valid in KALTRACK.)

2.3.1.4. STORE

STORE↵ This command stores the filtered card parameters out to the disk list called "KALLIST". The system asks for "LIST#". The system stores the card parameters in the location specified unless the specified number is greater than the final number in the list. In this case, the parameters are stored in the slot after the last card satellite in the list. Example:

```
*
STORE↵
LIST#.
1000↵
*              The newly created card parameters are stored in the last
               vacant position in the list because the list does not contain
               1000 satellites.
```

2.3.2. Retrieval

2.3.2.1. GETP

GETP↵ Get parameters from the parameter list and write them on the line printer and the system goes in a halt mode. Tracking can now be started by the CONT ↵ command. Example:

```
*
GETP↵
LIST NUMBER?  (I2)  The system wants to know the place in the list of
               parameters.
3↵             The operator wishes to track the object,
*              (star or satellite depending on the program in control) #3 in
               the list.
               Note:  This number is not the same as the star catalog number
               or the satellite number but only the number in order, in the
               list.
```

2.3.2.2. LIST

LIST↵ This command prints out the list resident in the core to the terminal without any more responses from the operator. (Not valid in KALTRACK.)

2.3.2.3. PRINT

PRINT↵ Print a listing of the mission to the line printer. For star track the print reads as follows: year, day, hour, minute, seconds, output azimuth, output elevation, azimuth read from the encoders, elevation read from the encoders, and number of outputs since the last print to the line printer.

For satellite track, the printout is: year, day, hour, minute, seconds, true anomaly, output azimuth, output elevation, azimuth read from encoders, elevation read from encoders, number of outputs since the last print to the LP, and number of clock errors since the last format to the LP.

The system also allows the operator to choose the print mode he wants, the print interval in seconds, and the number of prints per camera exposure. Example:

*	
PRINT↲	
INPUT PRINT MODE	System wants the print mode.
1 = NO PRINT	If any printing is being done, stop it.
2 = DISK PRINT	Write the data on the disk.
3 = $LPT PRINT	Print the data on the line printer.
4 = PRINT BOTH	Write on the disk and the LPT.
3↲	Operator chooses to print on the LPT.
PRINT INTERVAL	
10↲	Data is to be printed every ten seconds.
PRINT INTERVAL PER CAMERA QUEUE	
10000↲	This is a large number which indicates that no pictures are to be taken.
*	CLI

2.3.2.4. HEADER

HEADER↲	Output a header to the line printer just before the next print which must be less than or equal to eighty characters. Example:

*	
HEADER↲	Operator wishes to insert a header in the printout on the line printer.
HEADER:	System is ready to accept the message.
THIS IS A TEST.↲	Message to be inserted in the printout.
*	Message is inserted and control returned to CLI. System offsets of time and mount position are printed also.

2.4. MOUNT CONTROL

This section describes the commands used to direct the mount to desired orientations, either under operator control or under a track-routine control.

2.4.1. Operator-Specified Mount Control

2.4.1.1. MMOUNT

MMOUNT↲	Manually position the mount by using the TTY to add or subtract a preset set of increments to the mount offsets already in the system. If the mount is still under control of the command CMOUNT, it is overridden. There are subcommands available for the manual control. (Not valid in KALTRACK.)

A↲	Control the azimuth axis
E↲	Control the elevation axis
>↲ or .↲	Add a positive offset to the axis under control.
<↲ or ,↲	Add a negative offset to the axis under control.
?↲	Leave this mode and give the total offsets in degrees.
↲	Leave this mode without giving the offsets.

Any other character that is typed, is ignored. Example:

*	
MMOUNT	
AZ, EL SLIP INC. (D,M,S) PER < OR >	System axis for increment (decrement) value per increment (decrement) command.
3,30,0↲	Azimuth increment is 3.5 degrees.
5,15,0↲	Elevation increment is 5.25 degrees
E↲	Decrement/increment the elevation axis.
>↲	Add one positive increment.
.↲	Add one more increment.
.↲	Add one more increment.
,↲	Subtract one increment.
<↲	Subtract one more increment.
A↲	Decrement/increment the azimuth axis.
>↲	Add one offset
<↲	Subtract one offset.
,↲	Subtract one more.
<↲	Subtract one more.
<↲	Subtract one more.
.↲	Add one offset.
?↲	
AZ, EL OFFSETS IN DEGREES ARE - XX.XXXX XX.XXXX#	
*	CLI

2.4.1.2. CMOUNT

CMOUNT	This command achieves continuous mount positioning according to the increment rate per second supplied by the operator. Same subcommands as those under MMOUNT are available here also to facilitate tracking, but the increment (decrement) subcommands achieve slightly different results under this continuous mount positioning mode.
←↲ or 0↲	Reverse the direction of increment only for the axis under control.
<↲	Slow down the axis under control to half the current rate.
,↲	Slow down the axis under control to a third of the current rate.
>↲	Speed up the axis under control to twice the current rate.
.↲	Speed up the axis under control to thrice the current rate.
/↲	Stop the axis under control.
↲	System continues the operation but returns to CLI.
?↲	Leave this mode, print to the operator the total system offsets and stop any CMOUNT positioning activity.

These reported values are of total offsets and not only those due to the current MMOUNT command, if some offsets already existed before initiating MMOUNT.

If CLI is under control, and current mount positioning is to be altered, this mode is re-entered by typing CMOUNT which causes the axes movements to stop. If any character other than the above is typed,# then the system continues to move the mount at the latest chosen rate. Example:

```
*
CMOUNT
AZ, EL SLIP RATE (D,M,S/SEC)?      The system asks for the rate in (degrees,
                minutes, seconds)/sec. of time.
2,0,0           Azimuth rate would be 2°/sec.
1,0,0           Elevation rate would be 1°/sec.
E               Position the elevation.
L  #            Start the positioning at the given rate.
>               Double the elevation changing speed.
,               Reduce the elevation speed to a third.
0               Reverse direction of motion.
/               Stop elevation positioning.
A
L               Start azimuth positioning at the given rate.
,               Slow down to a third of the speed.
?               Stop azimuth change, give offsets, and return to CLI.
AZ, EL OFFSETS IN DEGREES ARE - XX.XXXXXXX     XX.XXXXX
*
```

2.4.1.3. INCM

```
INCM            Increment the previous mount offsets by the given amount.
                (Not valid in KALTRACK.) Example:
```

```
*
INCM
AZ, EL - INC (D,M,S)
2,3,4           Azimuth increment is 2°, 3 minutes, and 4 seconds.
4,5,6           Elevation increment is 4°, 5 minutes, and 6 seconds.
AZ OFFSET = X.XXXX   EL OFFSET = X.XXXX
*
```

2.4.1.4. SETM

```
SETM            Set the mount offsets to operator-specified new offsets by
                deleting previous offsets. Example:
```

```
*
SETM
AZ, EL (D,M,S)
3,2,4
5,6,7
*
```

In the example, since L is "any character other than above," it has the effect of starting the positioning at the given rate.

2.4.2. Track Routine Mount Control

In the previous subsection, mount control was achieved by mount offsets directly specified by the operator. The commands did not go through any tracking computations. Both commands in this subsection are valid for star track routine only. They may be used to center a star in the field of view by altering the RA and DEC values instead of AZ and EL angles.

2.4.2.1. INCS

INCS↲ This is a valid command in the star track program alone and it permits the operator to add or subtract an increment to the RA and DEC of the star already being tracked. The increment supplied does not alter the parameters in the list which can be changed only by the 'XFER' command. Example:

```
*
INCS↲
RA, DEC - INC (H,M,S   D,M,S)
4,6,8↲
6,8,9↲
RA OFFSET = XXX.XXXX   DEC OFFSET X.XXXX
*
```

2.4.2.2. SETS

SETS↲ This command also works for STARTRACK only, and it allows the operator to specify completely new RA and DEC values for the star being tracked without changing them in the parameter list. The mount starts tracking according to the new parameters. Example:

```
*
SETS↲
RA, DEC (H,M,S   D,M,S)
1,2,3↲
4,5,6↲
*
```

2.5. SEARCH COMMANDS

Strictly speaking, a search may be conducted by using any of the appropriate mount control commands described in the preceding subsection. However, the search process will then involve a tedious sequence of increments specified by the operator, and it is not always possible for the operator to select a correct set of increments. Especially for the case when the object is not visible in the field of view, a spiral search can be conducted by the commands described in 2.5.1. By experience, it has been determined that satellite tracking errors generally involve time offsets. That is, the calculated orientation of the mount may point slightly ahead or behind the actual satellite position in its orbit. If such is the case, the time slip commands described in subsection 2.5.2. will be invaluable.

2.5.1. Spiral Search (Not Valid in KALTRACK)

To use this option efficiently, the operator must have certain familiarity with the mechanism of search. A detailed description of the options is available in the section on acquisition and will not be repeated here.

2.5.1.1. SEARCH

SEARCH↵ — This command initiates the procedure to implement a spiral search in the line of sight coordinates with approximately constant linear speed. The system asks a number of questions to get a specification of the spiral to be drawn out about the calculated azimuth-elevation direction. In response to the questions, the operator must specify the view time, percentage of field of view overlap, and maximum angle of search. Example:

*
SEARCH↵
HELP — The system asks if the operator needs help.
0↵ — The operator declines. (No other option is yet operational.)
INPUT VIEW TIME IN SECONDS — This is the time for which a point on the spiral is required to stay in the field of view.
10↵ — View time is 10 seconds.
PERCENTAGE OF FIELD OF VIEW TO BE USED — This is the percentage of the aperture by which the spiral expands after going through 360°.
20↵ — Operator specifies 20% or 1 minute since the angular field of view is 5 minutes.
MAXIMUM RADIUS TO BE SEARCHED
5↵ — Radius is specified in terms of angle in 5 arc minutes.
TIME TO COMPLETE THE SPIRAL IS (IN SECONDS) 157.5762
$ — The $ sign indicates that the search CLI is in control instead of the main CLI, and the system will now be able to accept commands valid in the search routine. The spiral has now been initiated and the mount will continue to rotate about the continuously varying calculated position till it reaches the maximum specified limit or is interrupted by the commands to be described presently. The system does not respond with * since the main CLI is no longer in control and the search CLI is.

2.5.1.2. PAUSE

PAUSE↵ — This command halts the radius and angle changing at the instant the command was given and the mount continues tracking with the current offsets.

2.5.1.3. BACKTRACK

BACK↵ — This command allows the exact same spiral to be traced backward at an operator-specified rate, after the "PAUSE" was made in the search. The operator is asked to specify the backtrack

rate as a percentage of the rate, before PAUSE was issued. If no pause was issued, one is automatically inserted in the command stream. Example:

$	
BACK↵	
RATE	The system requires the backtrack rate.
50	The operator wants the system to describe the spiral backwards at half the previous rate.
$	

If the response was -100↵ instead of 50↵, the spiral would have continued to be described in the forward direction at the same rate as was before the PAUSE command. If a backtrack is allowed to continue to the center of the spiral, the search is automatically paused.

2.5.1.4. EXIT

EXIT↵ This command causes the system to exit the search, leaving the current offsets in the system and printing them out to the terminal in degrees.

2.5.1.5. ABORT

ABORT↵ The search is aborted and any offsets that were introduced during the search are nullified. Any offsets that existed before the search are left unchanged.

2.5.1.6. CSEARCH

CSEARCH↵ Any command, other than those described above, transfers the control to the main CLI with the existing offsets. The routine continues the spiral search by the CSEARCH command by returning to where the last spiral was left when the routine was exited. Any other commands than the above will cause an EXIT of this routine, but with the last specified search left in progress. This command allows the operator to re-enter after leaving.

2.5.2. Time Changing Commands

Commands under this subsection allow the operator to change the time for calculation of azimuth and elevation angles in a number of ways quite similar to the actual angle changes described in 2.4.1.

2.5.2.1. MSLIP

MSLIP↵ This command to manually change time is analogous to MMOUNT, which allows manual mount control. The symbol commands described in 2.4.1.1. under MMOUNT achieve similar effects on time offsets under MSLIP as they did on angle offsets under MMOUNT. (Not valid for KALTRACK.) Example:

```
*
MSLIP
SLIP PER  <OR>  ?  (SEC.)   The system asks for time slip in seconds per
                            symbol command ( < or> ).
.01
>                           Add one increment
<                           Subtract one increment
.                           Add one
,                           Subtract one
?                           Leave this routine
TIME OFFSET IN SEC = X.XX
*
```

2.5.2.2. CSLIP

CSLIP — This command for continuous change in time is analogous to CMOUNT, which allows continuous mount control. The symbol commands described in 2.4.1.2. under CMOUNT achieve similar effects on time slip rate in CSLIP as they did on the angle change rate under CMOUNT. Example:

```
*
CSLIP
SLIP RATE?  (SEC/SEC)
1
MAXIMUM TIME SLIP (SEC)
10
← or 0          Go backward in time
>               Speed up the time slip rate by a factor of 2
<               Slow down the time slip rate by a factor of 2
,               Go one third as fast
/               Pause
?               Exit and give total offset in time
TIME OFFSET IN SEC = X.XXXXX
*
```

2.5.2.3. INCT

INCT — Increment the time offset by a specified amount and type out the total offset. (Not valid in KALTRACK.) Example:

```
*
INCT
INC TIME OFFSET (H,M,S)
0,4,-3          Operator wants a time increment of 4 minutes and -3 seconds.
TIME OFFSET IN HOURS = .XXXXX H
*
```

2.5.2.4. SETT

SETT — New specified time offset is set and the previous offset is removed. Example:

```
*
SETT
TIME OFFSET (H,M,S)
1,2,3
*
```

2.5.2.5. TIME

TIME — This command simply prints out the current time from the clock without any offsets.

2.5.2.6. SDATE

SDATE — This command allows the operator to go back in time by typing a positive response to the system query. The command also clears any flags that have been mis-set by a time glitch from the external clock. The external clock often gives glitches which cause the date to advance by one day per glitch. SDATE corrects this problem. The command is issued after the operator has determined the number of days by which time has been shifted due to glitches. Such a condition generally causes a runaway mount which is immediately noticeable. Example:

```
*
SDATE
# OF DAYS TO GO BACK IN TIME?
2
*
```

2.5.2.7. SLIP

SLIP — This and subsequent commands may be considered subsets of CSLIP since what they achieve can be done equally well with CSLIP. SLIP initiates a time slip as in CSLIP. (Not valid in KALTRACK.) Example:

```
*
SLIP
SLIP RATE?  (SEC/SEC)
1             Change the time used in calculation by one second from current-
              ly used time per one second of time.
MAXIMUM TIME SLIP?  (SEC)
10
*
```

2.5.2.8. PAUSE

PAUSE↲ This command pauses a time slip already in progress. (Not valid in KALTRACK.)

2.5.2.9. BACKTRACK

BACK↲ This command allows a time slip in progress to be reversed at operator-specified speed similar to BACK in the spiral search 2.5.1.3. (Not valid in KALTRACK.) Example:

*
BACK↲
RATE
+100↲ Go back at the present rate
*

2.6. ALPHABETICAL INDEX OF COMMANDS

33	SETT	2.5.2.4.
34	SLIP	2.5.2.7.
35	START	2.2.2.7.
36	STARTRACK	2.2.2.3.
37	STOP	2.2.2.8.
38	STORE	2.3.1.4.
39	TIME	2.5.2.5.
40	WRITE	2.3.1.3.
41	XFER	2.3.1.1.

SYMBOL COMMANDS

42	>	2.4.1.1., 2.4.1.2., 2.5.2.1., 2.5.2.2.
43	<	2.4.1.1., 2.4.1.2., 2.5.2.1., 2.5.2.2.
44	?	2.4.1.1., 2.4.1.2., 2.5.2.1., 2.5.2.2.
45	.	2.4.1.1., 2.4.1.2., 2.5.2.1., 2.5.2.2.
46	,	2.4.1.1., 2.4.1.2., 2.5.2.1., 2.5.2.2.
47	/	2.4.1.2., 2.5.2.2.
48	←	2.4.1.2., 2.5.2.2.
49	0	2.4.1.2., 2.5.2.2.

SECTION 3

PASSIVE SATELLITE TRACK

3.1. INTRODUCTION

There are two aspects to the satellite tracking portion of the coelostat project. This section deals with the passive track by the coelostat mount. The track is passive in the sense that the program inputs are only the prespecified set of orbital parameters for an epoch and no real-time information about the satellite position is used for tracking. The sole real-time input for passive tracking is in the form of clock time and/or angle offsets described in the User's Manual. These offsets serve only to bring the satellite to the center of the field of view after acquisition. Active satellite track not only attempts to center the satellite but also updates the orbital elements, and is described in Section 5.

The passive satellite tracking program utilizes the general perturbation theory first proposed by Kozai [4] and simplified by the Space Defense Command (SDC). The Simplified General Perturbation (SGP) routine calculates the pointing angles, given the orbital parameters for a specified epoch [3]. The perturbations of six orbital elements of a close earth satellite moving in the gravitational field of the earth without air resistance are derived as functions of mean orbital elements and time. No assumptions about the order of magnitude of the eccentricity and inclination need be made, i.e., the trajectory solution is non-singular even at zero eccentricity and/or inclination.

Three types of perturbations have to be taken into consideration: 1) Secular, 2) Long-period and 3) Short-period.

3.1.1. Coordinate Systems

It is convenient to adopt the earth-centered inertial coordinate system (ECI) while tracking geocentric satellites. In this system, the earth's center is taken to be the origin. The x-axis points along the vernal equinox, which is the point of intersection of the projection of apparent path of sun on the earth's surface, and the equator where the sun goes from south to north in its apparent annual motion along the ecliptic. The y-axis lies in the equatorial plane and is perpendicular to the x-axis in a direction such that the z-axis, which completes the right-handed system points towards the North Pole (Figure 3-1). Polar coordinates in the ECI system are right ascension and declination angles as shown in the figure.

To define the orbital coordinate system it is first necessary to define the orbital parameters of a satellite moving in an elliptic orbit around the earth.

- a semi-major axis of the elliptic orbit in units of earth radii (ER).
- e eccentricity of the orbit
- i inclination of the orbital plane to the equatorial plane
- Ω right ascension of the ascending node, i.e., the point where the satellite crosses the equatorial plane going north from south.

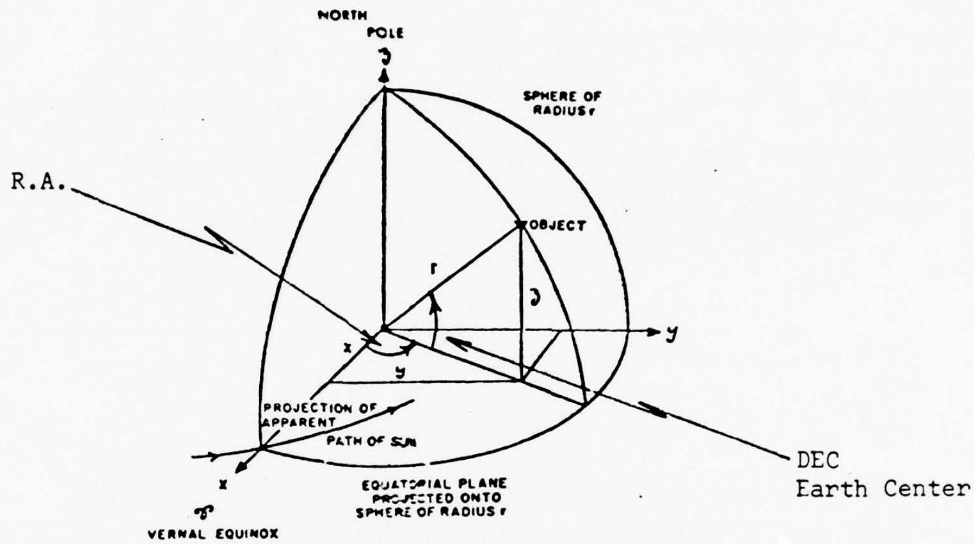


Figure 3-1 Earth-Centered Inertial Coordinate System

ω argument of perigee, i.e., the angle between the lines joining the earth center to the perigee (line of apsides) and the ascending node.

E true anomaly is the angle between the line of apsides and the radius vector pointing to the current satellite position.

In the orbital coordinate system, axis-ξ is taken along the line of apsides and ζ -axis is perpendicular to the orbital plane along the angular momentum vector. The η-axis is also in the orbital plane so as to make the system right-handed associated with the unit vectors i_ξ , i_η and i_ζ (Figure 3-2).

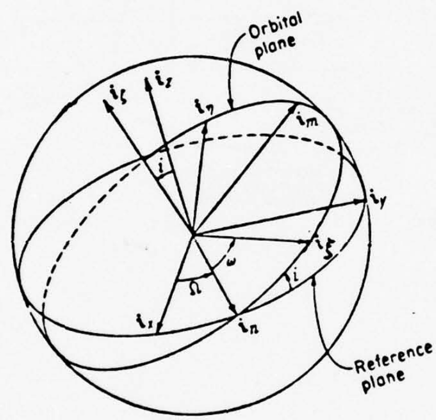


Figure 3-2 Orbital Coordinate System

The transformation of coordinates between ξ , η , ζ and the x,y,z axes is affected by means of the rotation matrix G_o.

$$\rho_{ECI} = G_o \rho_{OC} = \text{where} \tag{3.1-1}$$

$$G_o = \left[\begin{array}{c|c|c} C\,\Omega C\,\omega - S\,\Omega S\,\omega Ci & -C\,\Omega S\,\omega - S\,\Omega C\,\omega Ci & S\,\Omega Si \\ S\,\Omega C\,\omega + C\,\Omega S\,\omega Ci & -S\,\Omega S\,\omega + C\,\Omega C\,\omega Ci & -C\,\Omega Si \\ S\,\omega Si & C\,\omega Si & Ci \end{array}\right] \qquad (3.1\text{-}2)$$

where $C\theta = \cos\theta$ and $S\theta = \sin\theta$ and ρ_{ECI} and ρ_{OC} are the vectors in the ECI and Orbital coordinates respectively. The third coordinate system which will be required is the Local Topocentric System at the observatory site. In this system the E-axis is pointing along the local East, the N-axis along the local North and the Ze-axis completes the right-handed system by pointing along the local zenith. The transformation of coordinates between the x, y, z and the E, N, Ze axes is achieved by means of the following rotation matrix G_T at sidereal time θ at the time t,

$$\rho_T(t) = \begin{bmatrix} -S\theta & C\theta & 0 \\ -S\phi C\theta & -S\phi S\theta & C\phi \\ C\phi C\theta & C\phi S\theta & S\phi \end{bmatrix} \rho_I(t) \qquad (3.1\text{-}3)$$

i.e., θ = Greenwich hour angle + local latitude (λ_E) (Figure 3-3). $\rho_T(t)$, $\rho_I(t)$ are vectors at time t in the topocentric and inertial coordinates respectively, and ϕ is the latitude of the site, Figures 3-3, 3-4.

3.2. INITIALIZATION

The program is initialized using the two-card parameter set provided by SPACETRACK of the SDC. The format of data received is described in Table 3-1

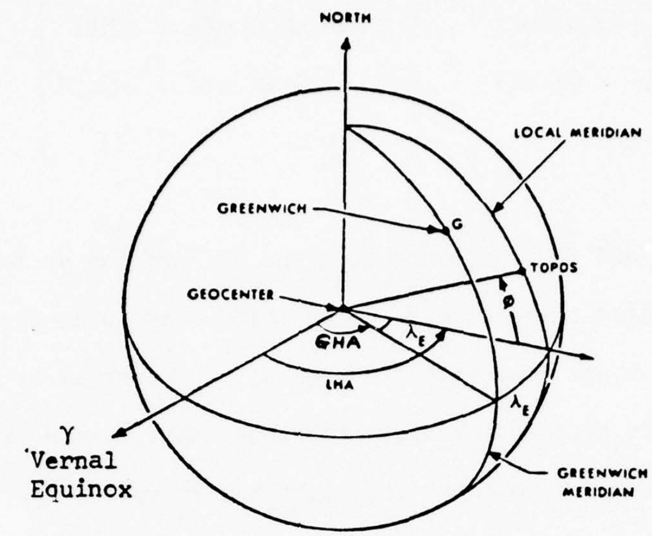


Figure 3-3 Geographic Coordinates

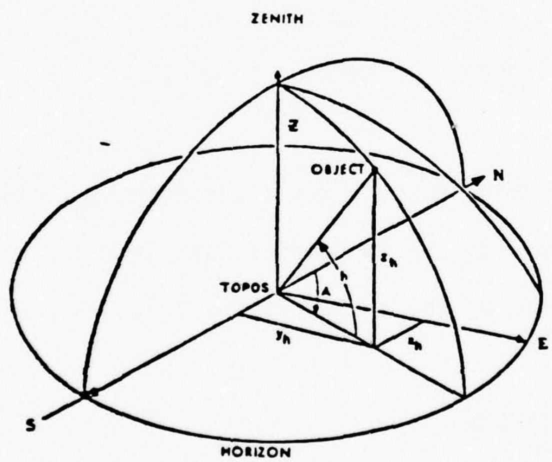


Figure 3-4 Local Topocentric, Horizon or Altazimuth System

CLASSICAL ELEMENT FORMAT (LINE ONE)

COL	NAME	DESCRIPTION	UNITS	FIELD FORMAT
1	LINNO	Line number of Element Data (Always 1 for Line 1)	None	X
2	Blank			
3-7	SATNO	Satellite Number	None	XXXXX
8	CLASI	Element Classification (U = Unclassified, C = Confidential, S = Secret)	None	X
9	Blank			
10-11	IDYR	International Designator (Last Two Digits of Launch Year)	Launch Yr	XX
12-14	IDLNO	International Designator (Launch Number of the Year)	None	XXX
15-17	IDPNO	International Designator (Piece of Launch)	None	XXX
18	Blank			
19-20	EPYR	Epoch Year (Last Two Digits of the Year)	Epoch Yr	XX

Table 3-1 NORAD Card Parameter Format

21-32	EPOCH	Epoch (Day and Fractional Days of the Year)	Days	XXX.XXXXXXXX
33	Blank			
34-43	NDOT2 or BTERM	First Time Derivative of the Mean Motion or Ballistic Coefficient (depending on the ephemeris type)	Revolutions per day^2 or $meters^2$ per kilogram	±.XXXXXXXX (If NDOT2 is greater than unity, a positive value is assumed w/o a sign)
44	Blank			
45-52	NDDOT6	Second Time Derivative of Mean Motion (Field will be blank if NDDOT6 is not applicable)	Revolutions per day^3	±XXXXX-X (Decimal point assumed between 45 and 46)
53	Blank			
54-61	BSTAR or AGOM	BSTAR drag term if GP4 general perturbations theory was used. Otherwise will be the radiation pressure coefficient.		±XXXXX-X
62	Blank			
63	EPHTYP	Ephemeris Type (Specifies the ephemeris theory used to produce the elements)	None	X
64	Blank			

Table 3-1 (Continued)

65-68	ELNO	Element Number	None	XXXX
69	CKSUM	Check Sum (Modulo 10)	None	X
		(LINE TWO)		
1	LINNO	Line Number of Element Data (Always 2 for Line 2)	None	X
2	Blank			
3-7	SATNO	Satellite Number	None	XXXXX
8	Blank			
9-16	II	Inclination	Degrees	XXX.XXXX
17	Blank			
18-25	NODE	Right Ascension of the Ascending Node	Degrees	XXX.XXXX
26	Blank			
27-33	EE	Eccentricity (decimal pt assumed)	None	.XXXXXXX
34	Blank			
35-42	OMEGA	Argument of Perigee	Degrees	XXX.XXXX
43	Blank			
44-51	MM	Mean Anomaly	Degrees	XXX.XXXX

Table 3-1 (Continued)

52	Blank			
53-63	NN	Mean Motion	Revolutions per day	XX.XXXXXXXX
64-68	REVNO	Revolution Number at Epoch	Revolutions	XXXXX
69	CKSUM	Check Sum (Modulo 10)	None	X

Table 3-1 (Continued)

and contains the necessary information about the satellite to be tracked, which includes:

1. epoch day and fractional day of the year,
2. angle of inclination of the orbit, i
3. right ascension of the ascending node, Ω
4. argument of perigee, ω
5. mean anomaly, M
6. mean motion, n, in revolutions per day
7. first time derivative of the mean motion, $\frac{\dot{n}}{2}$; and
8. second time derivative of the mean motion, $\frac{\ddot{n}}{6}$.

3.2.1. Constants

A number of constant parameters have to be specified either as data or in the initialization part of the program.

a_e Mean equatorial radius of earth = 6378.145 km

b_e Mean polar radius of earth = 6356.7598 km

f flattening of the earth = $\frac{a_e - b_e}{a_e} = \frac{1}{298.25} = 0.003352392$

GM Geocentric gravitational constant expressed as the product of the universal constant of gravitation (G) times the mass of

the earth (M) = 3.986012×10^{20} $cm^3 sec^{-2}$

= 11467.87425 $ER^3 DAY^{-2}$

= 0.0055304177 $ER^3 MIN^{-2}$

μ Mass function of earth = 1 $ER^3 k_e^{-2}$ MIN^{-2}

k_e Geocentric gravitational constant in SPACETRACK format

= 0.0743667785 $ER^{3/2}$ MIN^{-1} = 0.001239446 $ER^{3/2} SEC^{-1}$

$\equiv$ 107.0881622 $ER^{3/2} DAY^{-1}$

J_2 Second zonal harmonic coefficient of the geopotential function

= $1082.28 \pm .3 \times 10^{-6}$

J_3 Third zonal harmonic coefficient of the geopotential function

= -2.562×10^{-6}

λ Geodetic latitude of the telescope site (Verona) = 43.153291

Ø Longitude of the telescope site (Verona) = 75.625442W

= -75.625442

H Height above earth mean surface at the telescope (Verona)

= 495.54' or 2.368095852E-5 ER

3.2.2. Mean Values

The card data supplies the mean values of the parameters specified in the section titled Initialization. These parameters remain constant till they are updated, either by new card-parameter sets or by active tracking. These are the mean values at the specified epoch time and are subscripted "0". The mean values supplied to the program are used in calculating the following parameters not directly specified in the card format.

$$L_o = \text{mean orbital longitude}$$

$$= M_o + \Omega_o + \omega_o \tag{3.2-1}$$

The mean (Kozai) semi-major axis is calculated, taking into account the oblateness of the earth. For a satellite orbiting around a perfect sphere with an isotropic gravitational field we have the following relation between the semi-major axis of the ellipse and the orbital period of the satellite:

$$a^3 n^2 = \mu \tag{3.2-2}$$

where n is the mean motion in revolutions per day and μ=GM is the gravitational constant in units of ER^3DAY^{-2}. This gives the semi-major axis "a" in units of earth radii too. Due to the oblateness of the earth, the semi-major axis given by (3.2-2) has certain perturbation introduced into it which is a function of the orbit eccentricity and inclination.

$$a_o = a\,[1 + \frac{1}{3}\ \delta - \frac{1}{3}\,\delta^2] \tag{3.2-3}$$

where

$$\delta \triangleq -\ \frac{3}{2}\ J_2\ \frac{1}{a^2(1-e_o^2)^{3/2}}\ (1 - \frac{3}{2}\sin^2 i_o) \tag{3.2-4}$$

Now, a_o is the mean value of the semi-major axis in units of earth radii at the epoch, corresponding to the mean epoch values supplied by the card parameters. The derivatives of Ω and ω have been expressed as functions of a_o, e_o and i_o so that they result in imparting continuously varying perturbations to the satellite orbit, and are constant for a given set of parameters at an epoch.

$$\dot{\Omega}_o = -\ \frac{3}{2}\ J_2\ \frac{n_o}{a_o^2(1-e_o^2)^2}\ \cos i_o \tag{3.2-5}$$

$$\dot{\omega}_o = \frac{3}{2}\ J_2\ \frac{n_o}{a_o^2(1-e_o^2)^2}\ (2 - \frac{5}{2}\sin^2 i_o) \tag{3.2-6}$$

The variation of Ω is such that the plane of the orbit rotates about the earth's polar axis in a direction opposite to that of the motion of the satellite with the mean rate of rotation given by Equation 3.2-5. Similarly, $\dot{\omega}$ causes the line of apsides to rotate about the satellites angular momentum vector. It can be checked that if i_o >63° 26.7', the line of apsides will regress, while if i_o <63° 26.7', it will advance (Equation 3.2-6). $\dot{\omega}$ is negative in the former case and positive in the latter case.

3.3. SECULAR PERTURBATIONS

As explained in the preceding section, $\dot{\omega}_o$ and $\dot{\Omega}_o$ impart perturbations to the orbit which cause rotations of the orbital plane and the line of apsides about the polar axis. The first-order derivative of the mean motion, $\dot{n}$, is mainly the effect of a decaying orbit and is also responsible for secular perturbations of close earth satellites which experience appreciable drag effects. Secular perturbations are neither functions of ω nor M but of time. The time derivatives $\dot{\omega}_o$, $\dot{\Omega}_o$ and $\dot{n}_o$ appearing in the expressions for these perturbations are constants for a given epoch and therefore cause perturbations to be monotonous functions of time.

$$\Delta t \stackrel{\Delta}{=} t - t_o = \text{Time since epoch} \tag{3.3-1}$$

Since the mean anomaly M is defined as

$$M = n(t - t_o) \tag{3.3-2}$$

the change in mean anomaly since epoch due to the change in the mean motion is:

$$\Delta M = n_o \Delta t + \frac{\dot{n}_o}{2} \Delta t^2 + \frac{\ddot{n}_o}{6} \Delta t^3 \tag{3.3-3}$$

Similarly, $\dot{\omega}_o$ and $\dot{\Omega}_o$ cause the following changes:

$$\Delta\omega = \dot{\omega}_o \Delta t = \text{change in argument of perigee since epoch;} \quad (3.3\text{-}4)$$

$$\Delta\Omega = \dot{\Omega}_o \Delta t = \text{change in RA of ascending node since epoch.}$$

The intermediate mean values of the parameters which will be required for calculating the subsequent long and short-period perturbations are now calculated.

$$L_m = L_o + \Delta M + \Delta\Omega + \Delta\omega \quad (3.3\text{-}5)$$

where L is the mean orbital longitude defined in Equation 3.2-1.

$$\omega_m = \omega_o + \Delta\omega \quad (3.3\text{-}6a)$$

$$\Omega_m = \Omega_o + \Delta\Omega \quad (3.3\text{-}6b)$$

$$n_m = n_o + \left(\frac{\dot{n}_o}{2}\right) 2 \Delta t \quad (3.3\text{-}6c)$$

The semi-major axis is derived from Equation 3.2-2 as:

$$a_m = a_o \left(\frac{n_o}{n_m}\right)^{2/3} \quad (3.3\text{-}6d)$$

and

$$e_m = 1 - \left(\frac{a_o}{a_m}\right) (1-e_o) \quad (3.3\text{-}6e)$$

The equation for e_m is obtained by assuming that perigee does not change with time, thus: $q = a_o(1-e_o) = a_m(1-e_m)$. The inclination of the orbit does not suffer any secular perturbations [4].

3.4. LONG PERIODIC PERTURBATIONS

The long periodic perturbations, by definition [4], are functions of the argument of perigee. Since ω is a quantity with a period of variation much larger than the orbital period of the satellite, the perturbation terms which are functions of ω are said to be long periodic. The quantity ω itself undergoes secular perturbations due to $\dot{\omega}_o$ as explained in the preceding section. Hence, it causes the line of apsides to completely rotate about the satellite angular momentum vector in a period of time which is much larger than the orbital period. Therefore, perturbations dependent upon ω are said to be long periodic.

First, e_m and ω_m are transformed to a new set of variables

$$a_{xn} \triangleq e_m \cos \omega_m ; \; a_{yn} \triangleq e_m \sin \omega_m \qquad (3.4\text{-}1)$$

Then the effect of long periodic terms is calculated as follows:

$$\delta_L \triangleq \frac{J_3}{J_2} \frac{a_e}{a_m} \frac{\sin i_m}{(1-e_m^2)} \qquad (3.4\text{-}2)$$

$$a_{xn_L} = e_m \cos \omega_m$$

$$a_{yn_L} = e_m \sin \omega_m - \frac{1}{2}\delta_L \qquad (3.4\text{-}3)$$

$$\therefore \quad e_L = (a_{xn_L}^2 + a_{yn_L}^2)^{1/2} \qquad (3.4\text{-}4)$$

and $$\omega_L = \tan^{-1} \frac{a_{yn_L}}{a_{xn_L}} \qquad 0 \le \omega_L \le 2\pi \qquad (3.4\text{-}5)$$

The restriction of ω_L in 3.4-5 indicates that ω_L must be calculated as mod(2π). The long periodic term on L gives

$$L_L = L_m - \frac{1}{4}\delta_L a_{xn_L} \frac{3 + 5 \cos i_m}{1 + \cos i_m} \qquad (3.4\text{-}6)$$

$$0 \le L_L \le 2\pi$$

Now, Kepler's equation is solved for the eccentric anomaly using the function EXANM.

E_L = EXANM ($L_L - \omega_L - \Omega_m$, e_L) which solves $M = E - e \sin E$ by simple iteration

(3.4-7)

True argument of latitude:

$$u_L = 2\tan^{-1} \left\{ \left[\frac{1+e_L}{1-e_L}\right]^{1/2} \tan \frac{E_L}{2} \right\} + \omega_L \qquad (3.4\text{-}8a)$$

$$= v_L + \omega_L$$

Range: $R_L = a_m (1-e_L \cos E_L)$ (3.4-8b)

Semi-latus Rectum: $P_L = a_m (1-e_L^2)$ (3.4-8c)

Transverse component of velocity vector: $R\dot{v} = \frac{\sqrt{\mu P_L}}{R_L}$ (3.4-8d)

Radial component of velocity vector: $\dot{R} = \sqrt{\mu a_m} \frac{e_L}{R_L} \sin E_L$ (3.4-8e)

where v = true anomaly.

3.5. SHORT PERIODIC PERTURBATIONS

The short periodic perturbations are functions of the true argument of latitude u, which itself has the same period as the orbital period; hence, the perturbations are said to be short periodic.

$$\delta_S \triangleq \frac{1}{4} J_2 \left(\frac{a_e}{P_L}\right)^2 \quad (3.5\text{-}1a)$$

Range: $R_S = R_L + \delta_S(\sin^2 i_m \cos 2u) P_L$ (3.5-1b)

True argument of latitude: $u_S = u_L - \frac{1}{2} \delta_S (6-7 \sin^2 i_m) \sin 2u$ (3.5-1c)

Inclination: $i_S = i_m + 3 \delta_S \sin i_m \cos i_m \cos 2u$ (3.5-1d)

R.A. of
ascending node: $\Omega_S = \Omega_m + 3\,\delta_S \cos i_m \sin 2u$ (3.5-1e)

3.6. OSCULATING POSITION AND VELOCITY VECTORS

The new position and velocity vectors can now be calculated using the orbital parameters. The vector along the radial direction, $\underline{U}$ is given as follows:

$$\overline{U} = u_x\hat{i} + u_y\hat{j} + u_z\hat{k} \tag{3.6-1}$$

where

$$u_x = \cos u_S \cos \Omega_S - \sin u_S \sin \Omega_S \cos i_S \tag{3.6-2a}$$

$$u_y = \cos u_S \sin \Omega_S + \sin u_S \cos \Omega_S \cos i_S \tag{3.6-2b}$$

$$u_z = \sin u_S \sin i_S \tag{3.6-2c}$$

The unit vector $\underline{V}$, perpendicular to the radial vector, is along the direction of the transverse velocity component $R\dot{v}$ in the orbital plane.

$$V_x = -\sin u_S \cos \Omega_S - \cos u_S \sin \Omega_S \cos i_S \tag{3.6-3a}$$

$$V_y = -\sin u_S \sin \Omega_S + \cos u_S \cos \Omega_S \cos i_S \tag{3.6-3b}$$

$$V_z = \cos u_S \sin i_S \tag{3.6-3c}$$

The vector $\underline{W}$ which completes the triad is perpendicular to the orbital plane, i.e., parallel to the direction of the angular momentum vector.

$$\omega_x = \sin i_S \sin \Omega_S \tag{3.6-4a}$$

$$\omega_y = -\sin i_S \cos \Omega_S \tag{3.6-4b}$$

$$\omega_z = \cos i_S \tag{3.6-4c}$$

The components of the above unit vectors are along the earth-centered inertial coordinate system.

Now, the radius and velocity vectors can be written in terms of these vectors and their magnitudes.

$$R = R_S \underline{U} \tag{3.6-5a}$$

$$\underline{\dot{R}} = \dot{R}\,\underline{U} + R\dot{v}\,\underline{V} \tag{3.6-5b}$$

$$|\underline{\dot{R}}| = (\dot{R}_x^2 + \dot{R}_y^2 + \dot{R}_z^2)^{1/2} \tag{3.6-5c}$$

3.7. CALCULATION OF POINTING ANGLES

Site Vector: The site vector is obtained as a function of the sidereal time. The rate of rotation of earth is:

$$\dot{\theta} = 0.00007292116 \text{ rad./sec.}$$

The local sidereal time or hour angle (Figure 3-3) is

$$\theta_{LHA} = \theta_{GHA} - \lambda\omega$$

where θ_{GHA} is the hour angle at Greenwich meridian and $\lambda\omega$ the west longitude of the observer's meridian in degrees. If:

$$\theta_o = 99^\circ.6909833 + 36000^\circ.7689\ T_u$$

where

$$T_u = \frac{\text{J.D.} - 2415020.0}{36525} = \text{tropical centuries}$$

since noon January 0, 1900 and J.D. = Julian date which is obtainable from a calendar, then the Greenwich hour angle is expressed as

$$\theta_{GHA}(k) = \theta_o + \Delta t_k \frac{d\theta}{dt}$$

From this the recursion equation for the sidereal times at k and k+1 can be derived to be:

$$\theta_{(k+1)} = \theta_{(k)} + 7.292116 \times 10^{-5} \quad \Delta t_k$$

where Δt_k is in seconds, and $\Delta t_k = t_{k+1} - t_k$

$\emptyset$ = Geodetic latitude of the site

H = Height above sea level

f = Flatness ratio of earth = $\frac{a_e - b_e}{a_e}$ = 1/298.25

$$C \triangleq [1 - (2f-f^2)\sin^2\emptyset]^{-1/2} \quad ER$$

$$S \triangleq C[1-(2f-f^2)] \quad ER \tag{3.7-1}$$

where ER is the length unit, "Earth radius". Now, the site vector in the earth-centered inertial coordinates is calculated.

$$R_S(k+1) = \begin{bmatrix} (C+H)\cos\emptyset\cos\theta_{(k+1)} \\ (C+H)\cos\emptyset\sin\theta_{(k+1)} \\ (S+H)\sin\emptyset \end{bmatrix} \tag{3.7-2}$$

The vector from the site to the satellite in the inertial coordinates is:

$$\rho_I^{k+1} = R_{(k+1)} - R_S(k+1) \tag{3.7-3}$$

which is transformed to the topocentric system by the transformation matrix G_T in Equation 3.1-3.

$$\rho_T(k+1) = \begin{bmatrix} \rho_E \\ \rho_N \\ \rho_{Ze} \end{bmatrix}_{k+1} = \begin{bmatrix} -\sin\theta & \cos\theta & 0 \\ -\sin\emptyset\cos\theta & -\sin\emptyset\sin\theta & \cos\emptyset \\ \cos\theta\ \cos\emptyset & \cos\emptyset\sin\theta & \sin\emptyset \end{bmatrix}_{k+1} \rho_I(k+1) \tag{3.7-4}$$

The components ρ_E, ρ_N and ρ_{Ze} point towards the local (site) north, east and zenith respectively.

Calculation of azimuth (A) and elevation (h) angles from the vector $\rho_T(k+1)$ at time k+1 is then a trigonometric procedure (Figure 3-4).

$$A = \tan^{-1} (\rho_E / \rho_N) \tag{3.7-5}$$

and

$$h = \tan^{-1} \left\{ \rho_{Ze} \Big/ \sqrt{\rho_E^2 + \rho_N^2} \right\} \tag{3.7-6}$$

These values are given as input angles to the mount through the computer-mount interface.

3.8. DISCUSSIONS

The passive satellite tracking routine was used successfully to track a number of satellites. Some of the satellites were visible in the viewing telescope only and were brought to the center of the field of view using the mount control routines. Typically, the satellites are visible for no more than a few minutes (5-8 minutes at most) and sometimes are quite faint. A TV monitor must have a sensitive camera to be able to display them. Inflow of its position measurements to the active tracking program must also be as automated as possible to efficiently utilize the short time available for observation.

SECTION 4

ACQUISITION

4.1. INTRODUCTION

It may not always be possible to obtain up-to-date values of satellite orbital parameters, or the error in the parameters may be just large enough to cause the object being tracked to fall outside the field of view of the telescope. Therefore, if the object to be tracked is not visible in the scope, it would become necessary to search for it. Moreover, the nature of the error in angular position may be assumed to be a near-constant bias (in azimuth and elevation, see Figure 4-1) in any particular pass of the satellite. Such error appears as a constant offset from the direction pointed at by the telescope, so that a search can be conducted while the telescope is in the tracking mode. A spiral scan, performed while the "telescope mean direction" tracks the calculated satellite position, is the search method selected. Spiral search has some obvious advantages as compared to some other scan methods used in radar tracking; the methods being helical, Palmer, raster or nodding scan.

The spiral radius and angular position are both taken to be proportional to "square root of time from the spiral initialization." The "square-root-t" spiral, as it will be called henceforth, was chosen because it attains a constant linear velocity as the angle increases as opposed to the spiral where the radius and angle are some other functions of time. Since the radius and

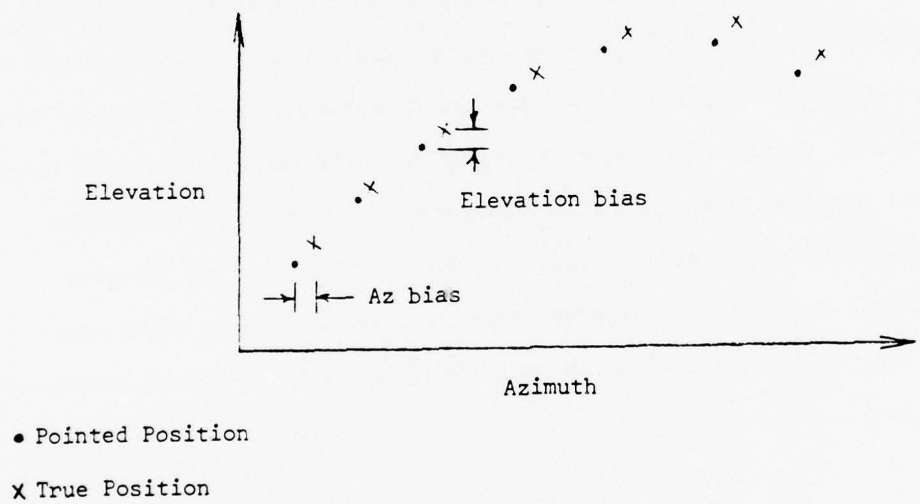


Figure 4-1 Nature of Tracking Error

the angle are both taken to be proportional to the same parameter, namely "square-root-t", an Archimedes spiral is obtained. The spiral of Archimedes (linear relation between radius and angle) gives a uniformly expanding spiral and all the desired "area" can be covered at an operator-specified speed. A search during the track mode will consist of the square-root-t spiral in the LOS (line-of-sight coordinate system to be defined in the next section) system and will appear to be a stretched and distorted spiral-like motion of the line of sight.

4.2. SPIRAL SEARCH BY THE LINE OF SIGHT (LOS)

Searching and tracking must be done simultaneously, which is to say that a spiral search about the calculated satellite position is to be performed while the mount is in the tracking mode.

Let us define the LOS coordinates such that the y-axis points along the LOS (when no spiralling is being attempted), the x-axis is perpendicular to the y-axis and parallel to the elevation mirror rotation axis. The z-axis completes the right-handed triad. When no search is being attempted, the LOS lies along the y-axis, i.e., along the direction specified by the instantaneous calculated values of the azimuth and elevation angles. The plane xz is parallel to the image plane with z-axis being the "vertical" and x-axis the "horizontal" (see Figure 4-2).

If (u_x, u_y, u_z) are unit vector components in the LOS system, then the corresponding vector (u_E, u_N, u_{ze}) in the local topocentric system is obtained

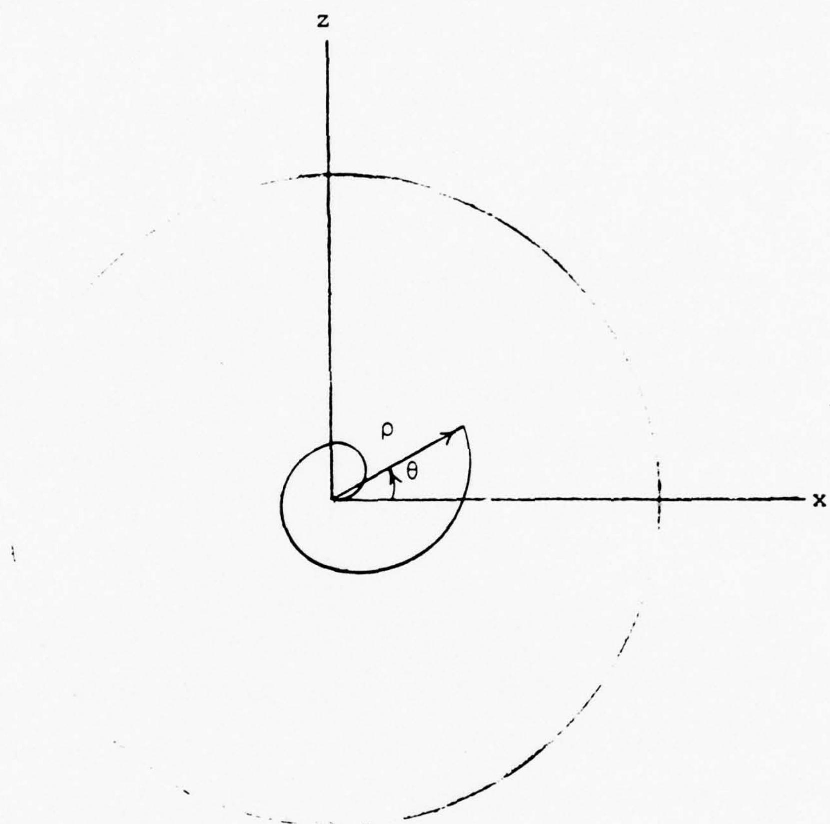


Figure 4-2 Line of Sight (LOS) Spiral in the xz-Plane

by the following transformation G_{LOS} (see Figure 4-3).

$$\begin{bmatrix} u_E \\ u_N \\ u_{Ze} \end{bmatrix} = \begin{bmatrix} \cos A & \cos h \sin A & -\sin h \sin A \\ -\sin A & \cos h \cos A & -\sin h \cos A \\ 0 & \sin h & \cos h \end{bmatrix} \begin{bmatrix} u_x \\ u_Y \\ u_z \end{bmatrix} \qquad (4.2\text{-}1)$$

Where A and h are the calculated azimuth and elevation angles, respectively.

Let $u_{LOS} = (u_x, u_y, u_z)^T$ be defined as a unit vector along the true LOS and u_{xz} its projection on the xz plane. The tip of the vector u_{xz} is required to describe an operator-specified spiral in the xz plane. The magnitude of u_{xz}, $\rho = |u_{xz}|$ must be allowed to grow as the $\arg(u_{xz})=\theta$ increases.

We have

$$u_x = \rho\cos\theta \quad ; \quad u_z = \rho\sin\theta$$

$$\therefore \quad u_y = \sqrt{1-u_x^2-u_y^2} = \sqrt{1-\rho^2} \qquad (4.2\text{-}2)$$

where ρ and θ are calculated as functions of time according to the specified square-root-t spiral. After transforming u_{LOS} to the topocentric system u_T, the new azimuth and elevation angles are computed from the following expressions.

$$\tan A' = \frac{u_E}{u_N} \; ; \; \tan h' = \frac{u_{Ze}}{\sqrt{u_n^2+u_E^2}} = \frac{u_{Ze}}{u_{NE}} \qquad (4.2\text{-}3)$$

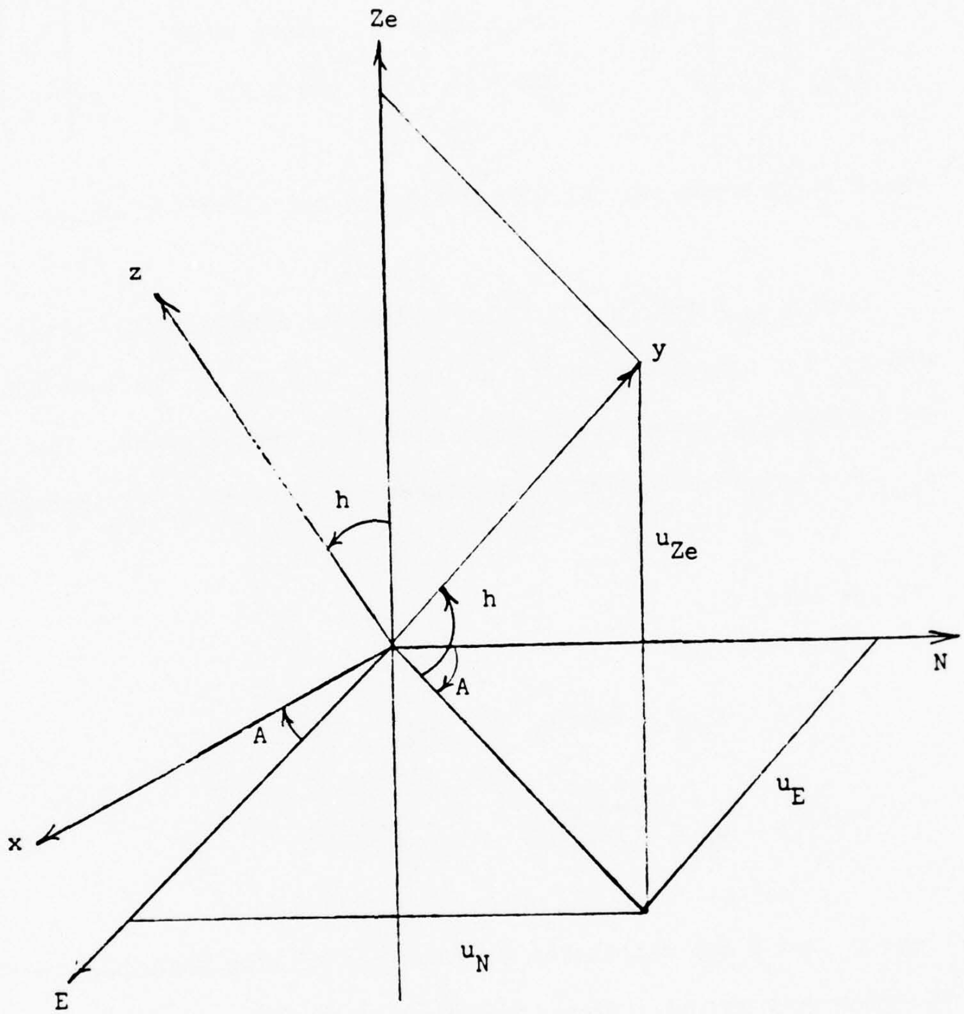


Figure 4-3 Topocentric and LOS Coordinates

The angles (A', h') are supplied to the mount, through the interface, as the pointing angles during the track-search mode, instead of the original A and h angles.

4.3. SPIRAL GENERATION

4.3.1. Spiral Parameters

A spiral is specified by the relation $\rho = f(\theta)$ or by specifying ρ and θ as functions of some common independent parameter. Now, spiral time is defined as "the time since the initiation of the spiral."

$$\text{Spiral time} = t = \text{Current time} - (\text{Time at } \rho = \theta = 0) \tag{4.3-1}$$

$$\rho \triangleq k_1\sqrt{t} \quad ; \quad \theta \triangleq k_2\sqrt{t} \tag{4.3-2}$$

$$\therefore \quad \rho = \frac{k_1}{k_2}\,\theta : \text{ Archimedes Spiral} \tag{4.3-3}$$

$$\frac{d\rho}{dt} = \frac{k_1}{2\sqrt{t}} \;;\; \frac{d\theta}{dt} = \frac{k_2}{2\sqrt{t}} = \frac{k_2}{k_1}\,\frac{d\rho}{dt} \tag{4.3-4}$$

Since the spiral is initiated with the object of searching, it will be advantageous to maintain constant the rate at which the center of the field of view moves along the spiral. This statement and the advantages of a square-root-t spiral will become clear presently.

The length of an element of the spiral between (ρ , θ) and (ρ + d ρ , θ +dθ) is given by,

$$ds = \sqrt{d\rho^2 + \rho\, d\theta^2} = \sqrt{\left(\frac{d\rho}{d\theta}\right)^2 + \rho^2}\; d\theta$$

$$= \sqrt{\frac{k_1^2}{k_2^2} + k_1^2\, t}\;\; \frac{k^2}{2\sqrt{t}}\; dt \qquad (4.3\text{-}5)$$

or

$$\frac{ds}{dt} = \frac{k_1 k_2}{2}\sqrt{\frac{1}{k_2^2 t} + 1} = \frac{k_1 k_2}{2}\sqrt{\frac{1}{\theta^2} + 1} \qquad (4.3\text{-}6)$$

From Equation 4.3-6 it is clear that the rate of change of the spiral length s approaches a constant with increase in time. If ρ and θ were proportional to t instead of $\sqrt{t}$, then $\frac{ds}{dt}$ would have increased rapidly, resulting in a decreasing "view time" and making viewing and object identification impractical after a certain time.

4.3.2. Spiral Specifications

4.3.2.1. View Time

The view time is defined as the time for which an object passing through the aperture center would remain in the field of view, and is determined by the spiral speed, $\frac{ds}{dt}$. The actual length of the field of view is a complicated function of the aperture diameter (AD) and the spiral parameters. However,

for $\theta > 2\pi$, the described spiral can be assumed to be fairly straight in parts, so that

Length of field of view $\cong$ AD , and

Time of view = T : Operator-specified quantity.

Let T be the view time at $\theta = 4\pi$, when $\frac{ds}{dt}$ has become fairly constant.

$$\therefore \left(\frac{ds}{dt}\right)_{\theta=4\pi} = \frac{k_1 k_2}{2}\sqrt{\frac{1}{16\pi^2} + 1} \cong \frac{AD}{T} \tag{4.3-7}$$

$$= \frac{k_1 k_2 \sqrt{16\pi^2+1}}{8\pi}$$

4.3.2.2. Overlap Fraction

The overlap fraction specified by the operator refers to the overlap of the apertures at θ and $\theta+2\pi$. The quantity is to be operator-specified and is the increase in the radius ρ as θ increases by 2π, in units of aperture diameter. From Equation 4.3-3 we have

$$\rho_{\theta+2\pi} - \rho_\theta = \frac{k_1}{k_2} 2\pi = f.AD \tag{4.3-8}$$

where f is the operator-specified fraction. Equations 4.3-7 and 4.3-8 can now be solved for the two unknowns k_1 and k_2. The aperture diameter is obtained from the angular field of view of the telescope, which is equal to 5 arc minutes. The AD for the unit vector considered in section 4.2. will be

$$AD = \text{angle x length from origin along LOS}$$

$$= \left(\frac{5}{60} \times \frac{\pi}{180}\right) \cdot 1 \qquad (4.3\text{-}9)$$

Using Equations 4.3-7 through 4.3-9 gives:

$$k_1 = 8.1928652 \times 10^{-4} \; (fT^{-1})^{1/2}$$

$$k_2 = 3.5393177 \; (Tf)^{-1/2} \qquad (4.3\text{-}10)$$

The third quantity that the operator must specify is the maximum angular deviation, ρ_{max}, to be searched in arc minutes. The program will calculate and output the search time.

$$t_{max} = \frac{\rho_{max}^2}{k_1^2} = \left(\frac{\rho_{max}}{60} \cdot \frac{\pi}{180}\right)^2 \frac{1}{k_1^2} \text{ sec.} \qquad (4.3\text{-}11)$$

4.4. OPTIONS AND CONTROL COMMANDS

4.4.1. Options

A number of options are included to aid in a search. The initialization options allow the operator to specify the following quantities.

a. The view time, which is the time taken by the aperture to traverse one aperture diameter along the spiral, is also the approximate time

that a point will stay in the field of view. A small view time will result in a fast spiral and vice versa.

b. The overlap fraction determines the compactness of the spiral and should be less than or equal to unity if no area around the calculated point is to be missed. Too small a value of f will result in a larger search time t_{max}.

c. Maximum search angle is determined by the expected error offset, and a value of 2-6 times the angular field of view should be quite suitable.

d. The search stops when ρ_{max} is reached and the mount continues along the latest offsets till a command to either backtrack or abort the search is issued.

e. The backtrack command retraces the spiral backwards at an operator-specified speed. It may be issued after the ρ_{max} is reached or after a pause command. The spiral can be stopped as soon as the object is viewed and the mount will continue at the latest offsets. If the aperture overshot the object, backtrack will be necessary. The operator may induce a slower backtrack so as not to miss the object again, and stop it as soon as the object reappears in the field of view.

4.4.2. Commands

The satellite track program has two command line interpreters (CLI). The main CLI deals with the total tracking program and the search routine CLI operates only when the program is in a search mode. At least the first four letters of the commands must be entered.

SEARCH: This command transfers the control to the search CLI even while the main program continues to track. The computer responds by asking for values of view time, aperture percentage overlay desired and the maximum angle of search. The operator is expected to input the view time in seconds, the fraction of the total aperture not overlapping after the spiral goes through 2π in terms of percentage of total aperture, and the maximum angle of search in arc minutes. When all the three questions are answered correctly by the operator, the program starts the spiral after putting out the expected time of search, depending on the maximum angle of search.

PAUSE: When the object is sighted, or for any other reason, the operator can issue this command to interrupt the search. The track continues by adding the latest offsets to the calculated azimuth and elevation values.

BACK: If the object is sighted during the search and it overshoots the center of field of view before issuing the PAUSE command, BACK command can be used to backtrack along the same spiral. The computer asks for the rate of backtracking in terms of percentage of the rate at which the original spiral was being

described. An operator response of a value between 0 and 100 causes a slower backtrack and a negative value causes the spiral to continue in the forward direction again.

EXIT: The EXIT command takes the task out of search-routine with the current azimuth and elevation offsets.

ABORT: This command aborts the search and zeros the existing azimuth and elevation offsets introduced by the search.

****: Any other command transfers the control to the main CLI.

CSEARCH: This command brings the control back to the search CLI.

4.5. RESULTS

The search routine has proved to be a valuable tool during tracking experiments. It has also been used to determine the fields of view of the main telescope and the viewing telescope on the television screen. By determining this correspondence, it is possible to observe the object on the TV screen and say whether it will be visible in any or both of the telescopes or not. It has also been used to determine the portion of the photographic film that is exposed through the main telescope. It has rarely been necessary to call the search routine to serve the purpose for which it has been designed. Stars are always visible in the center of the field of view because their RA

and DEC values are accurately tabulated in [3]. Time duration for which a satellite is above the horizon is small; however, most satellites were visible in the viewing telescope at the first attempt. Also, some of the satellites are either extremely dim and therefore barely visible, or blinking periodically due to spinning. Hence, it will be difficult for the viewer to make a decision to search for a satellite if it is not visible in the viewing scope. The invisibility could be attributed to large blinking period, small amount of reflected light or erroneous orbital parameters. Only in the latter case would it be advisable to search, and even then, sighting an acquired satellite cannot be guaranteed.

Any misalignment of the mount can be detected using the search. If a star is not visible in the center of the scope, the search routine can be used to acquire it. The offsets required to do so will be the mount misalignments.

SECTION 5

ACTIVE SATELLITE TRACK

5.1. INTRODUCTION

The passive satellite tracking routine is fed orbital elements of the satellite from the card parameters which have been calculated for some specified epoch. These parameters could have been calculated a few days earlier and therefore may differ slightly or substantially from the current values. After the satellite is acquired in the field of view, a number of measurements of its true azimuth and elevation can be obtained. These measurements can be used either on-line or off-line to update the orbital parameters.

The active tracking program was designed to take maximum advantage from the existing orbit prediction SGP routine described in the section on passive tracking. A model of the orbital parameter variation, based on the secular perturbations, was formulated. The filter is designed so as to accept azimuth and elevation angles as the observations.

5.2. LINEARIZED DISCRETE KALMAN FILTER

The theory of Kalman's recursive minimum variance filter has been described in innumerable papers[5] and texts[6]. Only a summary of steps required will be presented here.

There is a nonlinear relation between the states at adjacent time instants and also between the observations and the states (which in this case will be the orbital parameters).

System dynamics

$$X_{k+1} = f(X_k) + W_k \tag{5.2-1}$$

Observation equation

$$Z_{k+1} = h(X_{k+1}) + U_k \tag{5.2-2}$$

Where X_k is the six-dimensional state vector and Z_{k+1} is the two-dimensional observation vector. The nonlinear functions $f(X_k)$ and $h(X_k)$ are six-dimensional and two-dimensional vectors, respectively. In case of linear systems, they reduce to $F_k X_k$ and $H_k X_k$, where F_k and H_k are (6x6) and (2x6) matrices, respectively and independent of the state vector X_k.

The vector W_k is the system noise representing the uncertainty in the system model and is assumed to be Gaussian-distributed with zero mean and covariance Q_k. The measurement noise U_k is also assumed to be Gaussian-distributed with zero mean and covariance R_k.

$$E(W_k) = [0]_6 \quad ; \quad E(U_k) = [0]_2$$

$$E(W_k W_\ell) = Q_k \delta_{k\ell} \quad ; \quad E(U_k U_\ell) = R_k \delta_{k\ell} \tag{5.2-3}$$

Matrices Q_k and R_k must be positive semidefinite and definite, respectively.

Here, $\delta_{k\ell}$ is the Kroenecker delta function, emphasizing that there is no correlation in time in the noise sequences.

To initiate the filter, an initial estimate of the state, $\hat{X}_o$ and its covariance, P_o is required. This may be obtained from the preliminary knowledge about the orbit.

The predicted value of the state at a stage k+1 is obtained from the best prior estimate at stage k, $\hat{X}_k$,

$$\overline{X}_{k+1} = f(\hat{X}_k) \tag{5.2-4}$$

and the covariance prediction is obtained using the linearized state equation, which gives

$$F_k = \left[\frac{\partial f_k}{\partial X_k}\right]_{\overline{X}_{k+1}} = \left[f_{ij}\right]_{\overline{X}_{k+1}} \qquad i,j = 1,\ldots 6 \tag{5.2-5}$$

where the (6x6) matrix F_k is a linearized state matrix. If the predicted covariance is denoted by M_{k+1}, then

$$M_{k+1} \triangleq E\{(X_{k+1} - \overline{X}_{k+1})(X_{k+1} - \overline{X}_{k+1})^T\} = F_k P_k F_k^T + Q_k \tag{5.2-6}$$

The suboptimal Kalman gain for the linearized system is:

$$K_{k+1} = M_{k+1}H_{k+1}^T(H_{k+1}M_{k+1}H_{k+1}^T + R_{k+1})^{-1} \tag{5.2-7}$$

where M_{k+1} is the (2x6) linearized observation matrix.

$$H_{k+1} = \left(\frac{\partial h_k}{\partial X_{k+1}}\right)_{\overline{X}_{k+1}} = [h_{ij}]_{\overline{X}_{k+1}} \quad \begin{matrix} i = 1,2 \\ j = 1,\ldots 6 \end{matrix} \tag{5.2-8}$$

Both F_k and H_{k+1} have been linearized about the most recent estimate of the state, which in this case is $\overline{X}_{k+1}$. A residual is formed by differencing the observation and the predicted observation.

$$y_{k+1} = Z_{k+1} - h(\overline{X}_{k+1}) = \text{residual} \tag{5.2-9}$$

The final corrected estimate is:

$$\hat{X}_{k+1} = \overline{X}_{k+1} + K_{k+1}y_{k+1} \tag{5.2-10}$$

which has an error covariance matrix given by:

$$P_{k+1} = (I - K_{k+1}H_{k+1})M_{k+1} \tag{5.2-11}$$

where I is the (6x6) unity matrix. An equivalent and numerically more attractive form of Equation (5.2-11) is:

$$P_{k+1} = (I - K_{k+1}H_{k+1})M_{k+1}(I - K_{k+1}H_{k+1})^T + K_{k+1}R_{k+1}K_{k+1}^T \tag{5.2-12}$$

5.3. SYSTEM DYNAMICS

Six quantities are required to specify the position of a satellite in the ECI coordinate system. The six orbital parameters (or some of their

combinations) or the radius and radial velocity vectors are two of the common choices. The state vector to be estimated was chosen to be the set of six parameters because they undergo perturbation and have to be corrected as the observations are being taken. The state vector is therefore:

$$X_k^T = (a \quad e \quad i \quad \Omega \quad \omega \quad E)_k \tag{5.3-1}$$

where the subscript k refers to time t_k. The variation of this vector as a function of time is the system dynamics equation which is to be specified.

5.3.1. Recursion Equation

Since $a^3 n^2 = \mu$, and n is changing with time by virtue of $\dot{n}$, we can write the time derivative of a. This relation will be used often in this section, whenever a derivative of a function with respect to 'a' is required and the function contains the term 'n' - the mean motion.

$$\frac{da}{dt} = \mu^{1/3}(-\tfrac{2}{3})n^{-5/3}\dot{n}$$

$$\therefore \quad a_{k+1} = a_k + \dot{a}_k \Delta t_k$$

$$= a_k - \frac{4}{3} \frac{a_k}{n_k} \frac{\dot{n}}{2} \Delta t_k$$

$$= a_k\left(1 - \frac{4}{3} \frac{\dot{n}}{2} \frac{\Delta t_k}{n_k}\right) \tag{5.3-2}$$

Assuming that the perigee distance does not change with time, we have

$$q = a - ae = a(1 - e) = \text{Constant.} \tag{5.3-3}$$

Taking the derivative of Equation (5.3-3)

$$\frac{de}{dt} = \frac{1 - e}{a} \cdot \frac{da}{dt}$$

$$\therefore \quad \dot{e}_k = \dot{a}_k \frac{1-e_k}{a_k} = \frac{(1-e_k)}{\cancel{a}_k} \cdot \left(\frac{-4}{3}\right) \frac{\dot{n}}{2} \cdot \frac{\cancel{a}_k}{n_k}$$

$$= -\frac{4}{3} \frac{\dot{n}}{2} \frac{1-e_k}{n_k}$$

$$\therefore \quad e_{k+1} = e_k - \frac{4}{3} \frac{\dot{n}}{2} \frac{1-e_k}{n_k} \Delta t_k \tag{5.3-4}$$

Even though the inclination has short period perturbations, its mean value remains constant.

$$\therefore \quad i_{k+1} = i_k \tag{5.3-5}$$

The time derivatives of Ω and ω have been discussed in the Section 3.2.2. of passive tracking and they are:

$$\dot{\Omega}_k = -\frac{3}{2} J_2 \frac{n_k}{a_k^2(1-e_k^2)^2} \cos i_k$$

$$\dot{\omega}_k = +\frac{3}{2} J_2 \frac{n_k}{a_k^2(1-e_k^2)^2} \left(2 - \frac{5}{2} \sin^2 i_k\right)$$

$$\therefore \quad \Omega_{k+1} = \Omega_k + \dot{\Omega}_k \Delta t_k \qquad (5.3\text{-}6)$$

$$\text{and} \quad \omega_{k+1} = \omega_k + \dot{\omega} \Delta t_k \qquad (5.3\text{-}7)$$

The recursive equation for the eccentric anomaly can be derived from Kepler's equation.

$$M = E - e\sin E = n(t-t_o) \qquad (5.3\text{-}8)$$

$$E_{k+1} - E_k - e(\sin E_{k+1} - \sin E_k) = n_k \Delta t_k$$

$$\Delta E_k - e[\sin E_k + \Delta E_k \cos E_k - \sin E_k] = n_k \Delta t_k \qquad (5.3\text{-}9)$$

The above expansion is obtained under the assumption that Δt_k, and therefore, ΔE_k, are small.

$$\Delta E_k - e\cos E_k \cdot \Delta E_k = n_k \Delta t_k$$

$$\therefore \quad E_{k+1} = E_k + \frac{n_k}{(1-e\cos E_k)} \Delta t_k \qquad (5.3\text{-}10)$$

Equations (5.3-2 through 5.3-7 and 5.3-10) give the recursion of the state defined by Equation (5.3-1). These equations are used to get the predicted state $\overline{X}_{k+1}$ from the estimated state $\hat{X}_k$ as in Equation (5.2-4).

5.3.2. Linearized System Matrix

Since there is a nonlinear relation between X_{k+1} and X_k, linear Kalman filtering techniques cannot be directly applied. An approximation must be made and the first-order sensitivity matrix of X_{k+1} with respect to X_k must be derived[6]

$$F_k = \left[\frac{\partial f(X_k)}{\partial X_k}\right] = [f_{ij}] \tag{5.3-11}$$

where $$f_{ij} = \frac{\partial X^i_{k+1}}{\partial X^j_k} = \frac{\partial f^i_k}{\partial X^j_k} \tag{5.3-12}$$

From Equation (5.3-1), we have

$$X^1_k = a_k \;\; ; \;\; X^2_k = e_k \;\; , \;\; X^3_k = i_k \;\; ;$$

$$X^4_k = \Omega_k \;\; , \;\; X^5_k = \omega_k \text{ and } X^6_k = E_k \tag{5.3-13}$$

<u>1st Row:</u> Semi-major axis

$$f_{11} = \frac{\partial f^1_k}{\partial a_k} = 1 - \frac{4}{3}\,\frac{\dot{n}}{2}\,\frac{\Delta t_k}{n_k}$$

and $$f_{1j} = 0 \text{ for } j = 2 - 6 \tag{5.3-14}$$

2nd Row: Eccentricity

$$f_{21} = \frac{\partial f_k^2}{\partial a_k} = - \frac{\dot{n}}{n_k} \frac{(1-e_k)}{a_k} \quad \Delta t_k$$

$$f_{22} = \frac{\partial f_k^2}{\partial e_k} = 1 + \frac{4}{3} \frac{\dot{n}}{2} \frac{\Delta t_k}{n_k}$$

$$f_{23} = f_{24} = f_{25} = f_{26} = 0 \qquad (5.3\text{-}15)$$

3rd Row: Inclination

Since mean inclination is constant, all the terms in the third row of the matrix, except the (3x3) term, are zero.

$$f_{33} = \frac{\partial f_k^3}{\partial i_k} = 1$$

$$f_{31} = f_{32} = f_{34} = f_{35} = f_{36} = 0 \qquad (5.3\text{-}16)$$

4th Row: Right ascension of the ascending node is a function of all parameters except for the argument of the perigee and the eccentric anomaly.

$$\therefore \quad f_{45} = f_{46} = 0 \quad ; \quad f_{44} = 1$$

$$f_{41} = \frac{\partial f_k^4}{\partial a_k} = \frac{21}{4} J_2 \frac{n_k}{a_k^3 (1-e_k^2)^2} \cos i_k \quad \Delta t_k$$

$$f_{42} = \frac{\partial f_k^4}{\partial e_k} = -6\ J_2 \frac{e_k \quad n_k}{a_k^2(1-e_k^2)^3} \ \cos i_k \ \ \Delta t_k$$

$$f_{43} = \frac{\partial f_k^4}{\partial i_k} = +\frac{3}{2} J_2 \frac{n_k}{a_k^2(1-e_k^2)^2} \ \sin i_k \ \ \Delta t_k \qquad (5.3\text{-}17)$$

<u>5th Row</u>: Argument of Perigee is a function of all parameters except Ω_k and E_k.

$$\therefore \ f_{54} = f_{56} = 0. \ \text{Also } f_{55} = 1$$

$$f_{51} = \frac{\partial f_k^5}{\partial a_k} = -\frac{21}{4} \ J_2 \frac{n_k}{a_k^3 \ (1-e_k^2)^2} \ (2 - \frac{5}{2} \sin^2 i_k) \ \ \Delta t_k$$

$$f_{52} = \frac{\partial f_k^5}{\partial e_k} = 6\ J_2 \frac{n_k \quad e_k}{a_k^2(1-e_k^2)^3} (2 - \frac{5}{2} \sin^2 i_k) \ \ \Delta t_k$$

$$f_{53} = -\frac{15}{2} J_2 \frac{n_k}{a_k^2 \ (1-e_k^2)^2} \ \sin i_k \cos i_k \qquad (5.3\text{-}18)$$

<u>6th Row</u>: The eccentric anomaly is a function of a_k and e_k only, so that

$$f_{63} = f_{64} = f_{65} = 0.$$

$$f_{61} = \frac{\partial f_k^6}{\partial a_k} = -\frac{3}{2} \ \frac{n_k}{a_k} \ \frac{\Delta t_k}{(1-e_k \cos E_k)}$$

$$f_{62} = \frac{\partial f_k^6}{\partial e_k} = + \frac{n_k \cos E_k}{(1-e_k \ \cos E_k)^2} \ \ \Delta t_k \qquad (5.3\text{-}19)$$

$$f_{66} = \frac{\partial f_k^6}{\partial E_k} = 1 - \frac{n_k \ e_k \ \sin E_k}{(1-e_k \cos E_k)^2} \ \ \Delta t_k$$

Equations (5.3-14) through (5.3-19) define the (6x6) system dynamics matrix F_k which was introduced in Equation (5.3-11). The system noise matrix Q_k defined in Equation (5.2-3) is determined experimentally by trying different values in simulated missions. The system matrix F_k is calculated at $\overline{X}_{k+1}$; i.e., the predicted components of $\overline{X}_{k+1} = (\overline{a}_{k+1}, \overline{e}_{k+1}, \overline{i}_{k+1}, \overline{\Omega}_{k+1}, \overline{\omega}_{k+1}, \overline{E}_{k+1})^T$ are used to calculate the various elements of the system matrix. The calculation of F_k is simplified by the fact that out of a possible 36 terms, only twelve are non-zero terms which require recalculation at every computation stage, and three more are equal to unity.

The state matrix F_k is used to calculate the predicted covariance M_{k+1} according to Equation (5.2-6) from the error covariance P_k at the preceding stage.

5.4. OBSERVATIONS

The azimuth and elevation angles will be the observations for the updating of the orbital elements. There is a non-linear relationship between the observations and the orbital elements of the form given by Equation (5.2-2) which must be explicitly determined to be able to derive the linearized observation matrix given by Equation (5.2-8).

5.4.1. Relation Between Angle Measurements and the Orbital Elements

The radius vector to the satellite in the orbital coordinates is given by:

$$\bar{r}_{k+1} = \begin{bmatrix} a_{k+1}(\cos E_{k+1} - e_{k+1}) \\ a_{k+1}(1 - e_{k+1}^2)^{1/2} \sin E_{k+1} \\ 0 \end{bmatrix}_{\bar{X}_{k+1}} \tag{5.4-1}$$

This vector is now transformed to the ECI coordinates using the transformation matrix G_o from Equation (3.1-1)

$$G_o = G_\Omega \, G_i G_\omega \tag{5.4-2}$$

where

$$G_{\Omega_{k+1}} = \begin{bmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix}_{k+1} \tag{5.4-3a}$$

$$G_{i_{k+1}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{bmatrix}_{k+1} \tag{5.4-3b}$$

and $$G_{\omega_{k+1}} = \begin{bmatrix} \cos\omega & -\sin\omega & 0 \\ \sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{bmatrix}_{k+1} \qquad (5.4\text{-}3c)$$

Therefore, the satellite vector in the ECI coordinates is:

$$\overline{R}_{k+1} = G_o \underline{r}_{k+1} \qquad (5.4\text{-}4)$$

so that $\overline{R}_{k+1}$ is calculated at $\overline{X}_{k+1}$. From here onwards to the calculation of the predicted azimuth and elevation angles, the computations are identical to those from subsection 3.7. on calculation of pointing angles.

If $R_{S_{k+1}}$ represents the site vector in the ECI coordinates, and G_T the transformation matrix in Equation (3.7-4), then the site-satellite vector in the local topocentric coordinate system becomes:

$$\rho_T(k+1) = G_T(R_{k+1} - R_{S_{k+1}}) = G_T\,\rho_I(k+1)$$

$$= G_T[G_o\,\underline{r}_{k+1} - R_{S_{k+1}}] \qquad (5.4\text{-}5)$$

where $\rho_T(k+1) = \begin{bmatrix} \rho_E \\ \rho_N \\ \rho_{Ze} \end{bmatrix}_{k+1}$ has components

along the local east, north, and zenith. The measured angles are then:

$$\begin{bmatrix} A \\ \\ \\ h \end{bmatrix}_{k+1} = \begin{bmatrix} \tan^{-1} \left(\dfrac{\rho_E}{\rho_N} \right) \\ \\ \tan^{-1} \left(\dfrac{\rho_{Ze}}{\sqrt{\rho_E^2 + \rho_N^2}} \right) \end{bmatrix} = h(X_{k+1}) \qquad (5.4\text{-}6)$$

The actual measured angles would be the true angles given by this equation, contaminated by the instrument measurement errors which are represented by Equation (5.2-2).

5.4.2. Linearized Observation Matrix

In Equation (5.4-5), $\rho_I(k+1)$ is the site-satellite vector in the ECI coordinates. The derivative of the expression for $h(X_{k+1})$ with respect to X_{k+1} has to be taken by the application of the chain rule.

$$\frac{\partial h(X_{k+1})}{\partial X_{k+1}} = \frac{\partial h}{\partial \rho_I(k+1)} \cdot \frac{\partial \rho_I(k+1)}{\partial X_{k+1}} = \frac{\partial h}{\partial \rho_I(k+1)} \cdot \frac{\partial}{\partial X_{k+1}} (R-R_S)_{k+1} \qquad (5.4\text{-}7)$$

Considering that R_S is independent of X_{k+1}, since it is the site vector, Equation (3.7-2), and also suppressing the subscript (k+1) for simplicity, we get:

$$H = \frac{\partial h(X)}{\partial X} = \frac{\partial h}{\partial \rho_I} \cdot \frac{\partial R}{\partial X} = \frac{\partial h}{\partial \rho_T} \cdot \frac{\partial \rho_T}{\partial \rho_I} \cdot \frac{\partial (G_o \underline{r})}{\partial X}$$

$$= H_1 H_2 \qquad (5.4\text{-}8)$$

where,

$$H_1 \overset{\Delta}{=} \frac{\partial h}{\partial \rho_I} = \text{a (2x3) matrix}$$

$$H_2 \overset{\Delta}{=} \frac{\partial R}{\partial X} = \text{a (3x6) matrix}$$

so that H is a (2x6) matrix.

5.4.2.1. Calculation of $H_1 = [h'_{ij}]$

The derivative of the vector h, with respect to the vector ρ_I, is obtained simply by considering that the derivative of arc tangent:

$$\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

First row:

$$h'_{11} = \frac{\partial A}{\partial \rho_X} = \frac{\rho_N^2}{\rho_N^2 + \rho_E^2} \frac{d}{d\,\rho_X} \left(\frac{\rho_E}{\rho_N}\right) \qquad (5.4\text{-}9)$$

where ρ_x, ρ_y, ρ_z are the components of ρ_I in the ECI system. The expressions for (ρ_E ρ_N ρ_{Ze}) in terms of (ρ_x ρ_y ρ_z) are obtained from Equation (3.7-4).

$$h'_{11} = \frac{\rho_N^2}{\rho_N^2 + \rho_E^2} \cdot \frac{d}{d\,\rho_x} \left\{ \frac{-\sin\theta\rho_x + \cos\theta\rho_y}{-\sin\phi\cos\theta\rho_x - \sin\phi\sin\theta\rho_y + \cos\phi\rho_z} \right\}$$

$$= \frac{\rho_N^2}{\rho_N^2 + \rho_E^2} \cdot \frac{-\sin\theta \rho_N + \rho_E \sin\phi \cos\theta}{\rho_N^2}$$

$$= \frac{-\rho_N \sin\theta + \rho_E \sin\phi \cos\theta}{\rho_N^2 + \rho_E^2} \qquad (5.4\text{-}10a)$$

Proceeding in a similar fashion, other terms of the matrix H' can be derived to give the following expressions.

$$h'_{12} = \frac{\partial A}{\partial \rho_y} = \frac{\rho_N \cos\theta + \rho_E \sin\phi \sin\theta}{\rho_E^2 + \rho_N^2} \qquad (5.4\text{-}10b)$$

$$h'_{13} = \frac{\partial A}{\partial \rho_Z} = \frac{-\rho_E \cos\phi}{\rho_E^2 + \rho_N^2} \qquad (5.4\text{-}10c)$$

Second row:

Let $\rho_E^2 + \rho_N^2 + \rho_{Ze}^2 = \rho_{ZEN}^2$ and $\rho_E^2 + \rho_N^2 = \rho_{EN}^2$

$$h'_{21} = \frac{\partial h}{\partial \rho_x} = \frac{\rho_E^2 + \rho_N^2}{\rho_E^2 + \rho_N^2 + \rho_{Ze}^2} \left(\frac{\rho_{EN} \cos\theta \cos\phi - \frac{\rho_{Ze}}{2} \frac{2(\rho_E + \rho_N)(-\sin\theta - \sin\phi \cos\theta)}{\rho_{EN}}}{\rho_{EN}^2} \right)$$

$$= \frac{\rho_{EN}^2 \cos\theta \cos\phi + \rho_{Ze}(\rho_E + \rho_N)(\sin\theta + \sin\phi \cos\theta)}{\rho_{ZEN}^2 \rho_{EN}} \qquad (5.4\text{-}10d)$$

Similarly:

$$h'_{22} = \frac{\partial h}{\partial \rho_y} = \frac{\rho_{EN}^2 \cos\phi \sin\theta - \rho_{ZE}(\rho_E + \rho_N)(\cos\theta - \sin\phi \sin\theta)}{\rho_{ZEN}^2 \rho_{EN}} \qquad (5.4\text{-}10e)$$

$$h'_{23} = \frac{\partial h}{\partial \rho_Z} = \frac{\rho^2_{EN} \sin\phi - \rho_{ZE}(\rho_E + \rho_N)\cos\phi}{\rho^2_{ZEN}\ \rho_{EN}} \tag{5.4-10f}$$

Equations (5.4-10) define the matrix H_1 completely.

5.4.2.2. Calculation of H_2

The Jacobian of the transformation (5.4-4) is the matrix H_2 which is obtained by writing R_{k+1} explicitly in terms of the orbital parameters [Equations (5.4-1) through (5.4-4)] and then taking appropriate partial derivatives. Since the relation is available as a matrix-vector product in Equation (5.4-4), H_2 can be calculated columnwise by taking derivative of R_{k+1} with respect to each of the orbital elements.

<u>1st Column:</u> Only r_s is a function of the semi-major axis a.

$$\frac{\partial R}{\partial a} = G_o \frac{\partial r_s}{\partial a} = G_o \begin{bmatrix} \cos E - e \\ \sqrt{1-e^2}\ \sin E \\ 0 \end{bmatrix} \tag{5.4-11a}$$

<u>2nd Column:</u> Only r_s is a function of the eccentricity e.

$$\frac{\partial R}{\partial e} = G_o \frac{\partial r_s}{\partial e} = G_o \begin{bmatrix} -a \\ -ae(1-e^2)^{-1/2}\sin E \\ 0 \end{bmatrix} \qquad (5.4\text{-}11b)$$

3rd Column:

$$\frac{\partial R}{\partial i} = \frac{\partial}{\partial i} G_\Omega G_i G_\omega r_s = G_\Omega \frac{\partial G_i}{\partial i} G_\omega r_s$$

$$\frac{\partial R}{\partial i} = G_\Omega \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin i & -\cos i \\ 0 & \cos i & -\sin i \end{bmatrix} G_\omega r_s \qquad (5.4\text{-}11c)$$

4th Column:

$$\frac{\partial R}{\partial \Omega} = \frac{\partial G_\Omega}{\partial \Omega} G_i G_\omega r_s = \begin{bmatrix} -\sin\Omega & -\cos\Omega & 0 \\ \cos\Omega & -\sin\Omega & 0 \\ 0 & 0 & 0 \end{bmatrix} G_i G_\omega r_s \qquad (5.4\text{-}11d)$$

5th Column:

$$\frac{\partial R}{\partial \omega} = G_\Omega G_i \frac{\partial G_\omega}{\partial \omega} r_s = G_\Omega G_i \begin{bmatrix} -\sin\omega & -\cos\omega & 0 \\ \cos\omega & -\sin\omega & 0 \\ 0 & 0 & 0 \end{bmatrix} r_s \qquad (5.4\text{-}11e)$$

6th Column:

$$\frac{\partial R}{\partial E} = G_\Omega G_i G_\omega \quad \frac{\partial r_s}{\partial E} = G_o \begin{bmatrix} -a \sin E \\ a\sqrt{1-e^2} \ \cos E \\ 0 \end{bmatrix} \qquad (5.4\text{-}11f)$$

Now, the linearized measurement matrix H is given by postmultiplying H_1 by H_2, Equation (5.4-8).

The measurement error covariance matrix R is taken to be positive definite and compatible with the expected error variance in the angle-measuring instruments. Generally, it is taken to be a diagonal matrix. Then the Kalman gain, the corrected estimate of the six orbital parameters and the updated covariance matrix are computed according to Equations (5.2-7), (5.2-10) and (5.2-11) respectively. The whole process described in Sections 5.3 and 5.4 is repeated at each observation sampling stage.

SECTION 6

PROGRAM DESCRIPTION

6.1. INTRODUCTION

This section of the report describes the computer program and hardware interface developed by PAR for driving the optical tracking mount at the PATS tower of the Verona test annex of the RADC. The theory supporting the tracking programs has been described in Sections 3, 4, and 5 of this report. The current section gives a brief description of the FORTRAN coded programs, at varying levels of detail, to supplement the elaborate documentation provided in the computer source listing in the form of comments. The listing of each routine includes a definition of each of the internal and external variables, common blocks, required inputs and available outputs, detailed description of the subroutine function, operations performed in different sections of the program and external subprograms.

The system has been described with supporting flowcharts of varying complexities from overall system blocks to individual subroutine descriptions. Then, a short description of the hardware is followed by an index of programs.

The main functions that the overall system can perform are:

a. Track stars given the right ascension and declination from star catalogs;

b. Track satellites given NORAD satellite card parameters using a passive tracking routine;

c. Track a satellite and update its parameters by filtering its observed azimuth and elevation positions using the Kalman filtering technique.

All tracking systems have a hard-wired clock pulse input for the Greenwich Mean time and other inputs required, depending upon the tracking program in control. The STARTRACK program requires the right ascension, declination, brightness magnitude and a catalog number for the star to be tracked for its STARLIST. However, only the first two quantities are used for the actual tracking calculations. The track is accurate to within a few arc seconds and the star is always observed in the center of the field of view of the main telescope, as long as current RA and DEC values from the catalog are supplied.

The SATTRACK routine is a passive satellite tracking program which requires the two-card parameter set provided by NORAD as an input. It uses the Simplified General Perturbation (SGP) theory to calculate the instantaneous satellite position. The satellite offsets from the telescope center vary from satellite to satellite and depend on the accuracy of the card parameters.

The mount control commands described in Section 2.4. of the User's Manual can then be used to introduce offsets in the calculated mount position to bring the satellite to the center of the field of view. Sometimes the offset is large enough for the satellite not to be visible in the main scope, which has a field of view of only 5 arc minutes, but it is easily visible in the viewing telescope which has a 1° field of view. None of the satellites that were tracked caused the mount to slew; i.e., the speed of the satellite was well within the capability of the mount.

The KALTRACK routine is the active tracking program which requires a number of approximate azimuth and elevation angles of the satellite to be supplied manually at sequential time instants. The program uses these input measurements to update the satellite parameters. In a follow-up contractual work, the necessity to input manually will be replaced by hardware. Since updating of parameters is possible only after taking repeated measurements, the initial parameter set with which the program is started must be accurate enough for the satellite to be visible in the telescope.

6.2. SYSTEM DESCRIPTION

This section describes the system with the aid of flowcharts at different levels of complexity.

6.2.1. Overall System

Figure 6-1 is a flowchart of the operations required to bring up the Real-time Disk Operating System (RDOS) on the NOVA 800 computer. A further detailed description is available in the User's Manual to complement the flowchart.

Depending on the tracking operation to be performed, the user can type STARTRACK, SATTRACK or KALTRACK to read the appropriate program into the core, Figure 6-2. The program in core, after getting the appropriate parameters and commands, generates azimuth and elevation angles as functions of time and transmits to the mount through the mount-computer hardware interface.

6.2.2. File Maintenance

Each of the three tracking systems (STARTRACK, SATTRACK and KALTRACK) maintains a data file on the objects tracked frequently. The command LIST (2.3.2.2.) may be used to retrieve the STARLIST or SATLIST as the case may be. Modification of the list resident in the core is possible by reading in cards. Using the RDOS command CARDINPUT↲ for SATTRACK or KALCARD for KALTRACK, a reorganized and updated listing of the list is printed on the line printer for either command (Figure 6-3, 6-4). To obtain a printout of the KALLIST, it is necessary to use the command KALCARD↲ with maximum number of cards as zero. KALLIST cannot be obtained while the tracking is in progress, as opposed to STARLIST and SATLIST which can be obtained during track by using the command LIST ↲ . The STARLIST can accomodate 100 star parameters. Each set of parameters for a star consists of the catalog number of the star, brightness magnitude, right ascension in hours, minutes, seconds

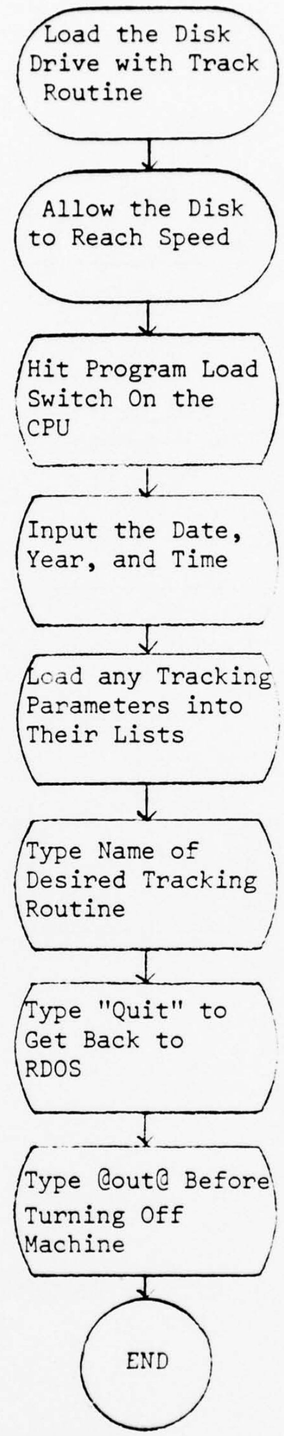


Figure 6-1 Bringing Up the System

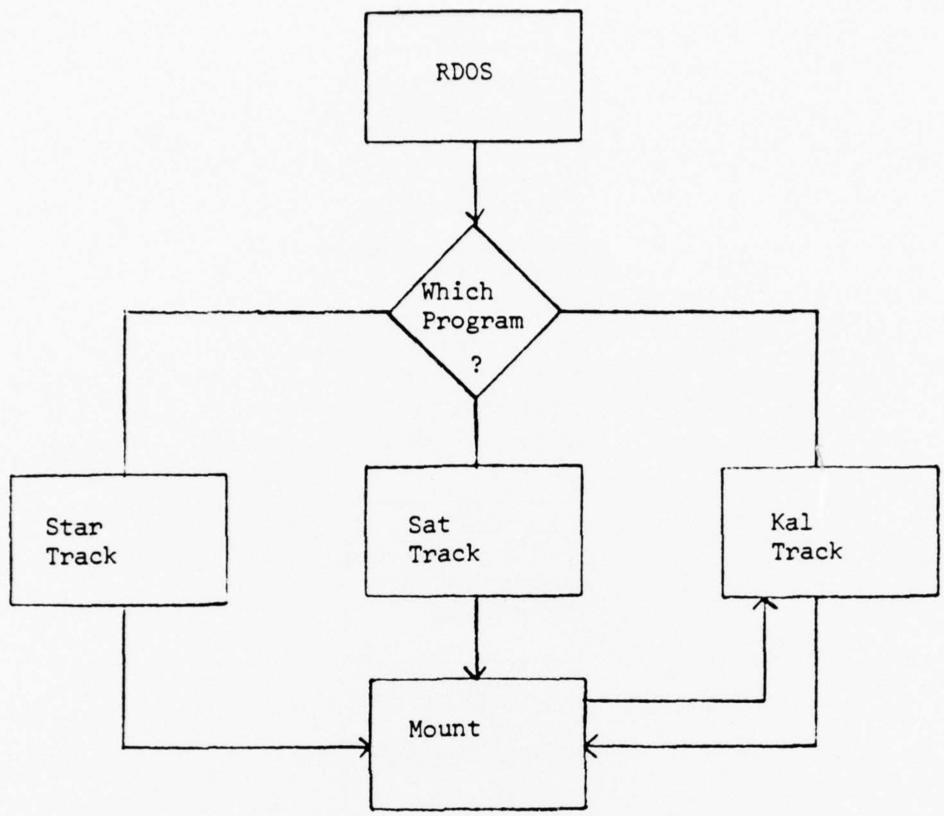


Figure 6-2 Optical Tracking System

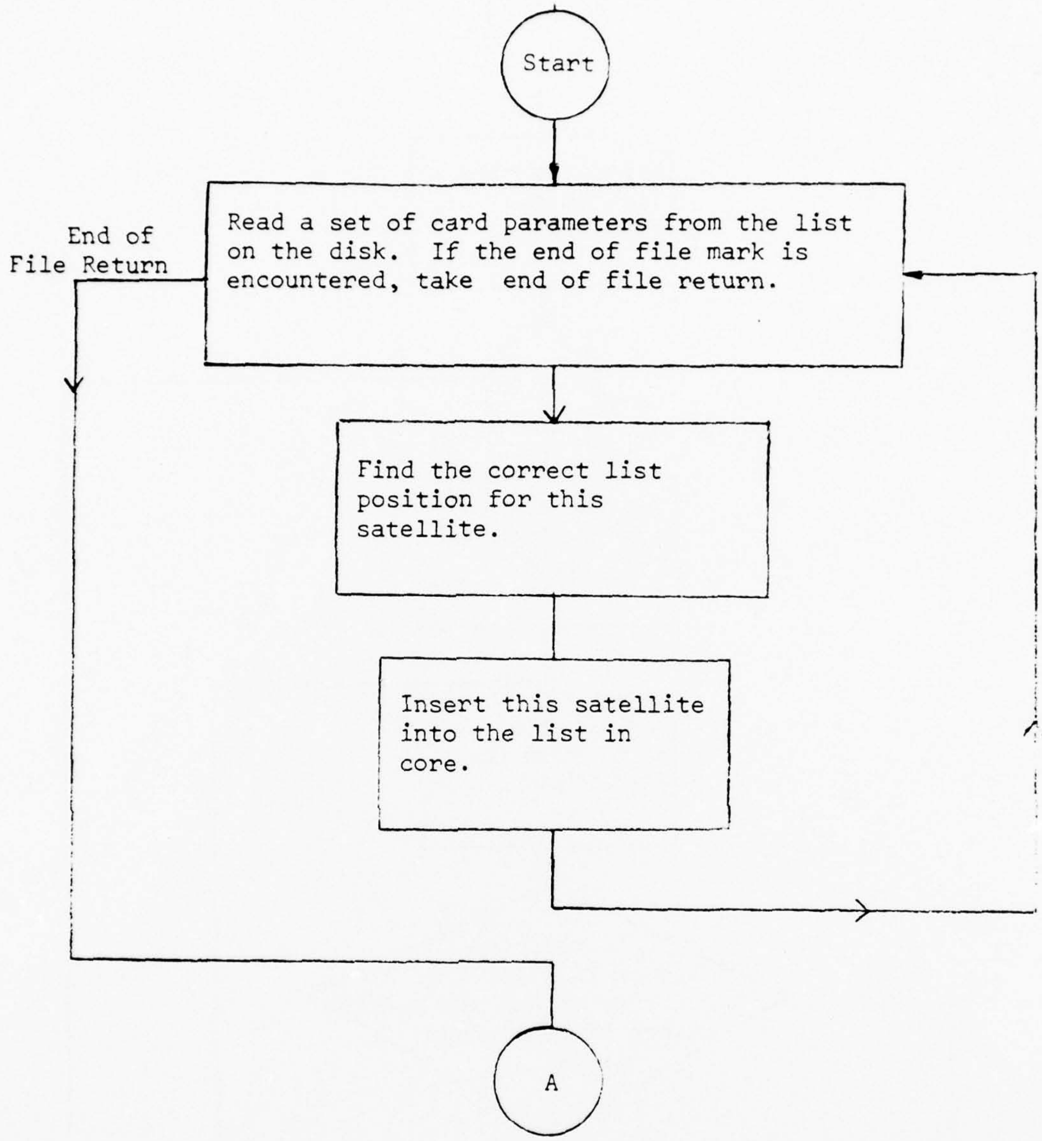


Figure 6-3 Flow Chart for Card Input

Figure 6-3 (Continued)

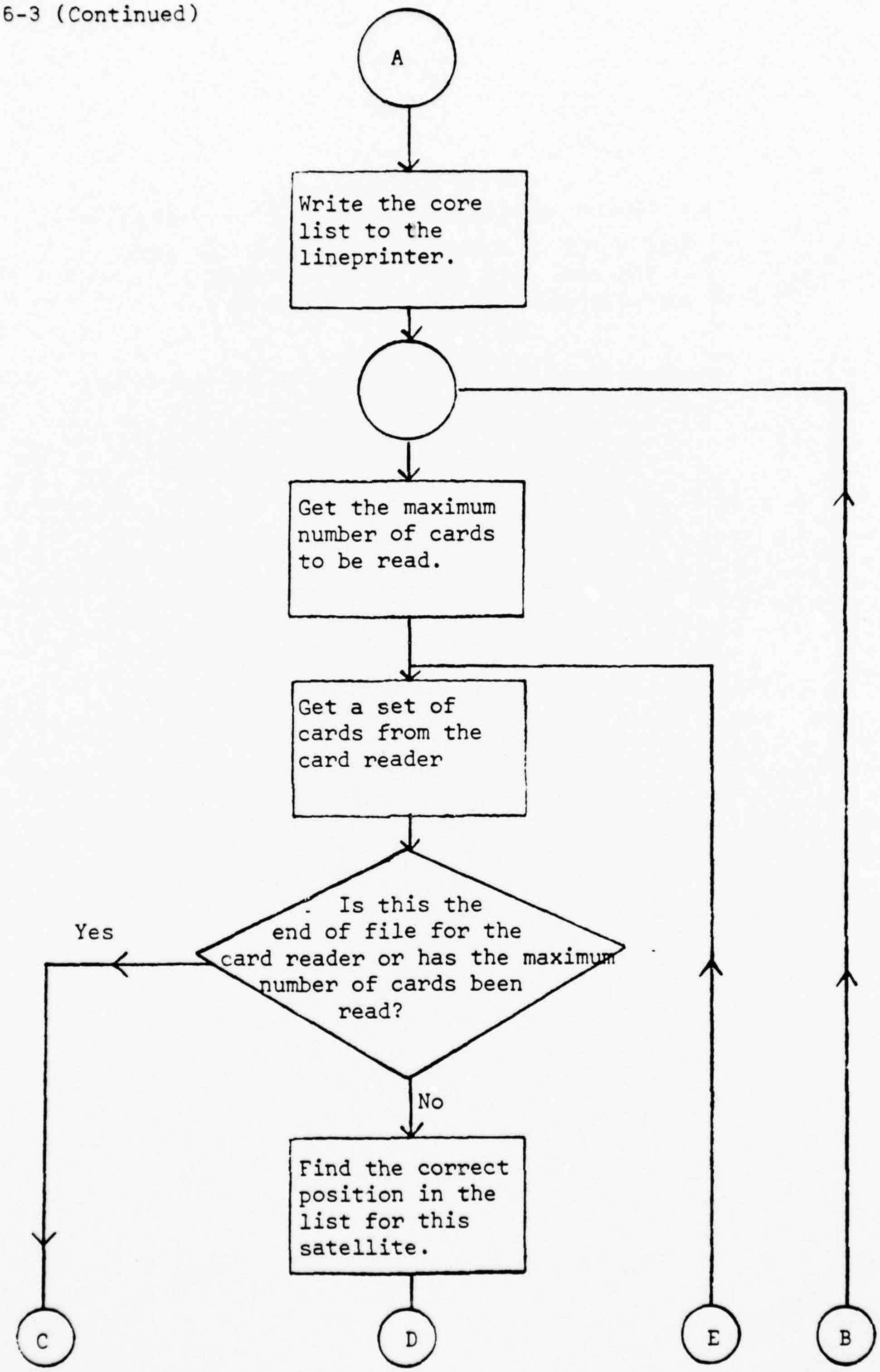
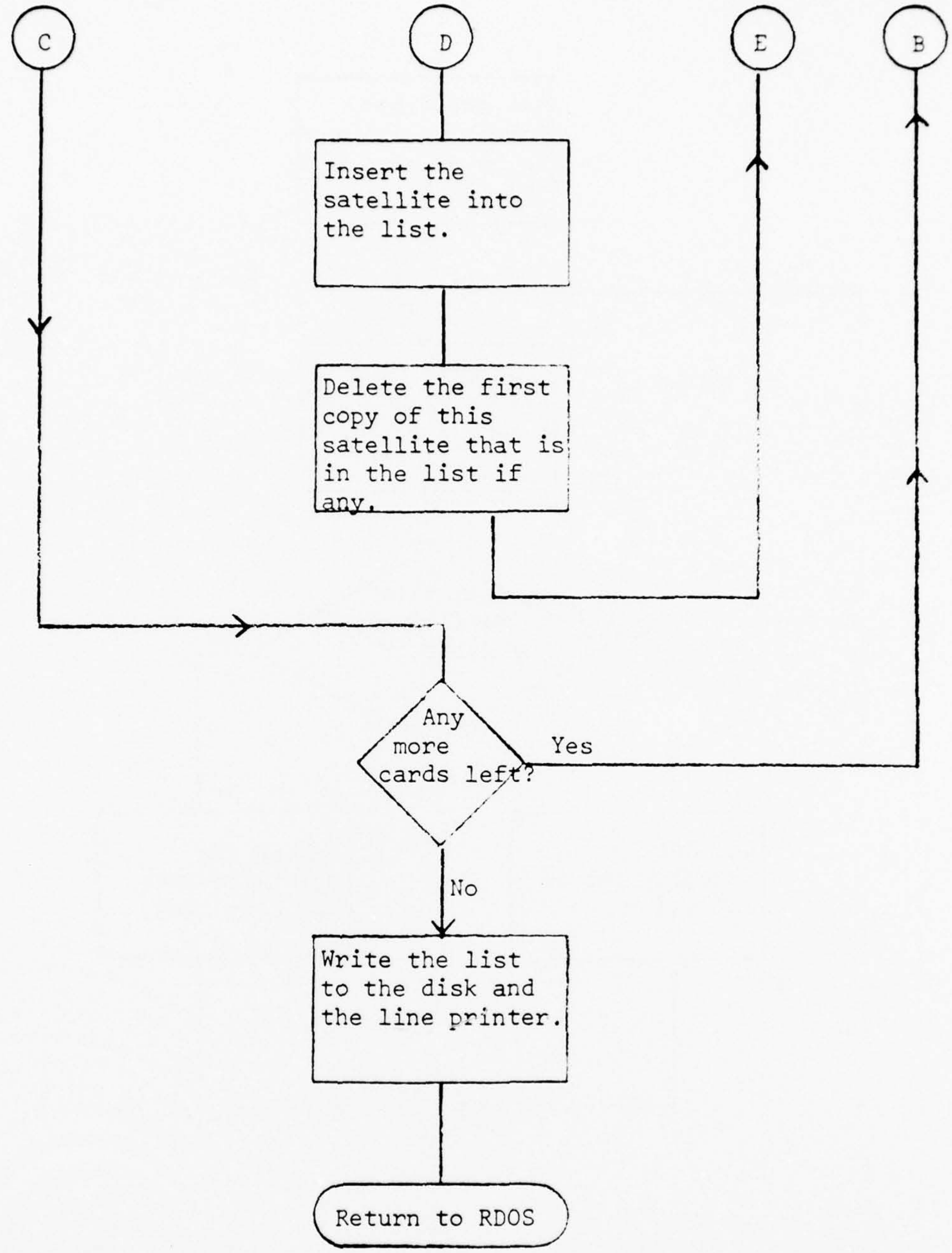


Figure 6-3 (Continued)



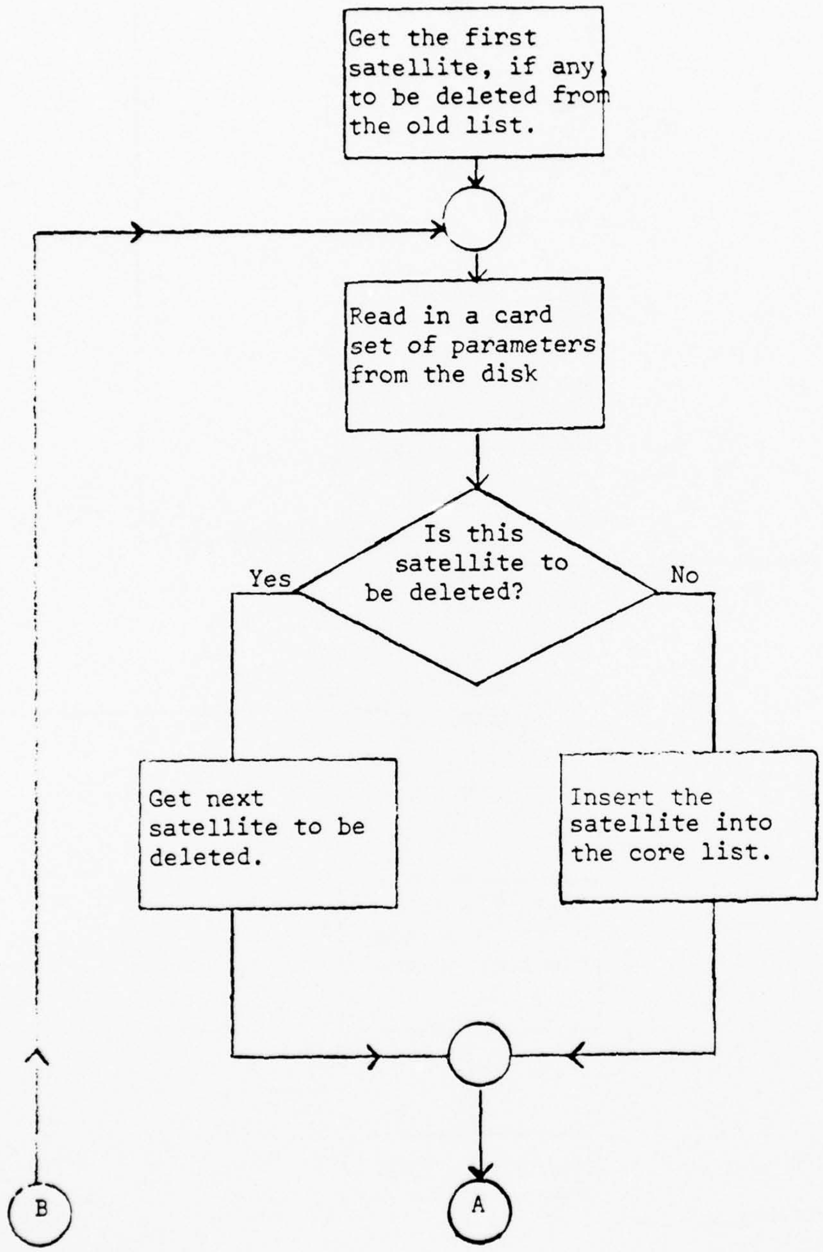


Figure 6-4 Flowchart for Kalcard

Figure 6-4 (Continued)

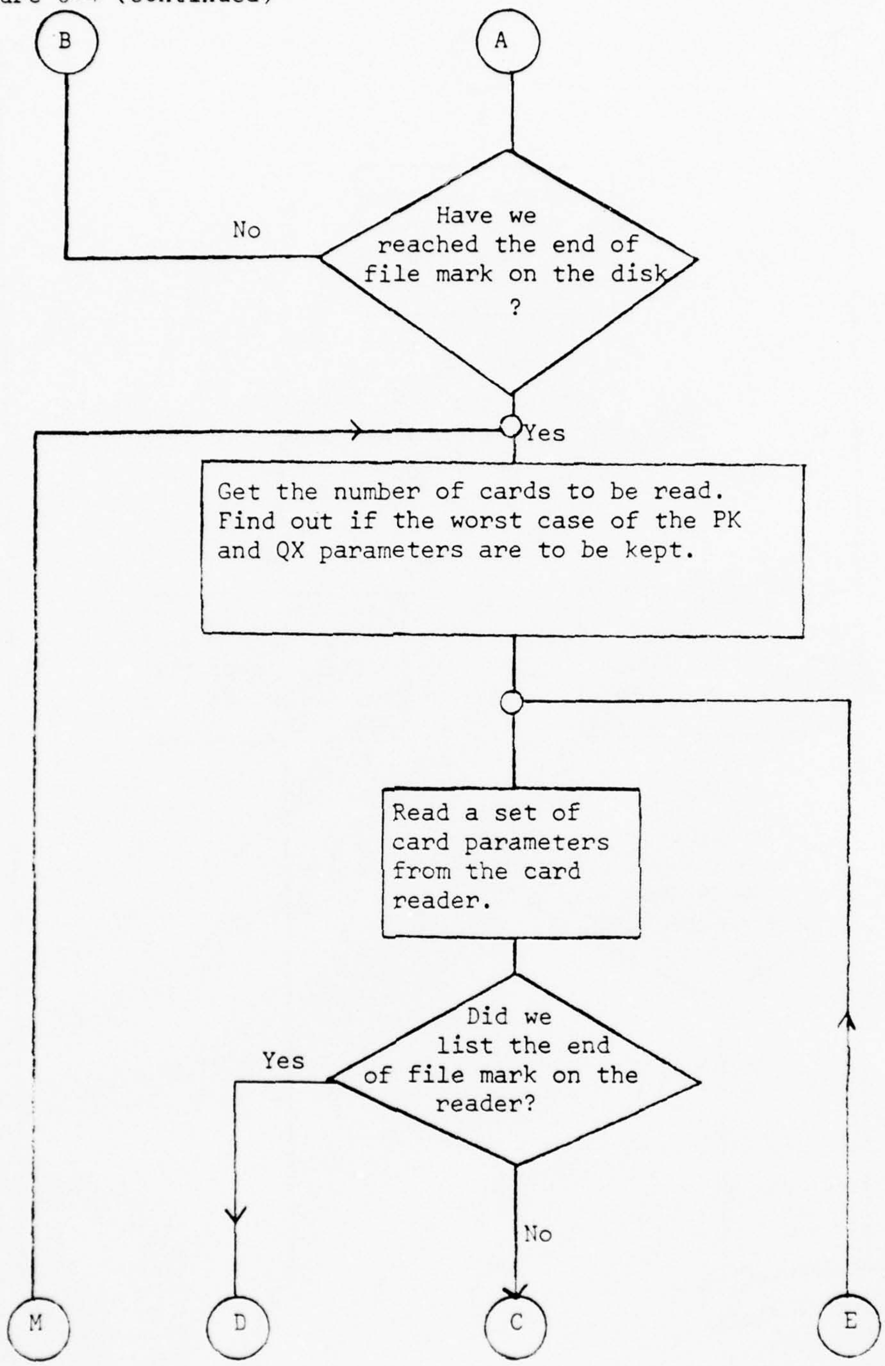


Figure 6-4 (Continued)

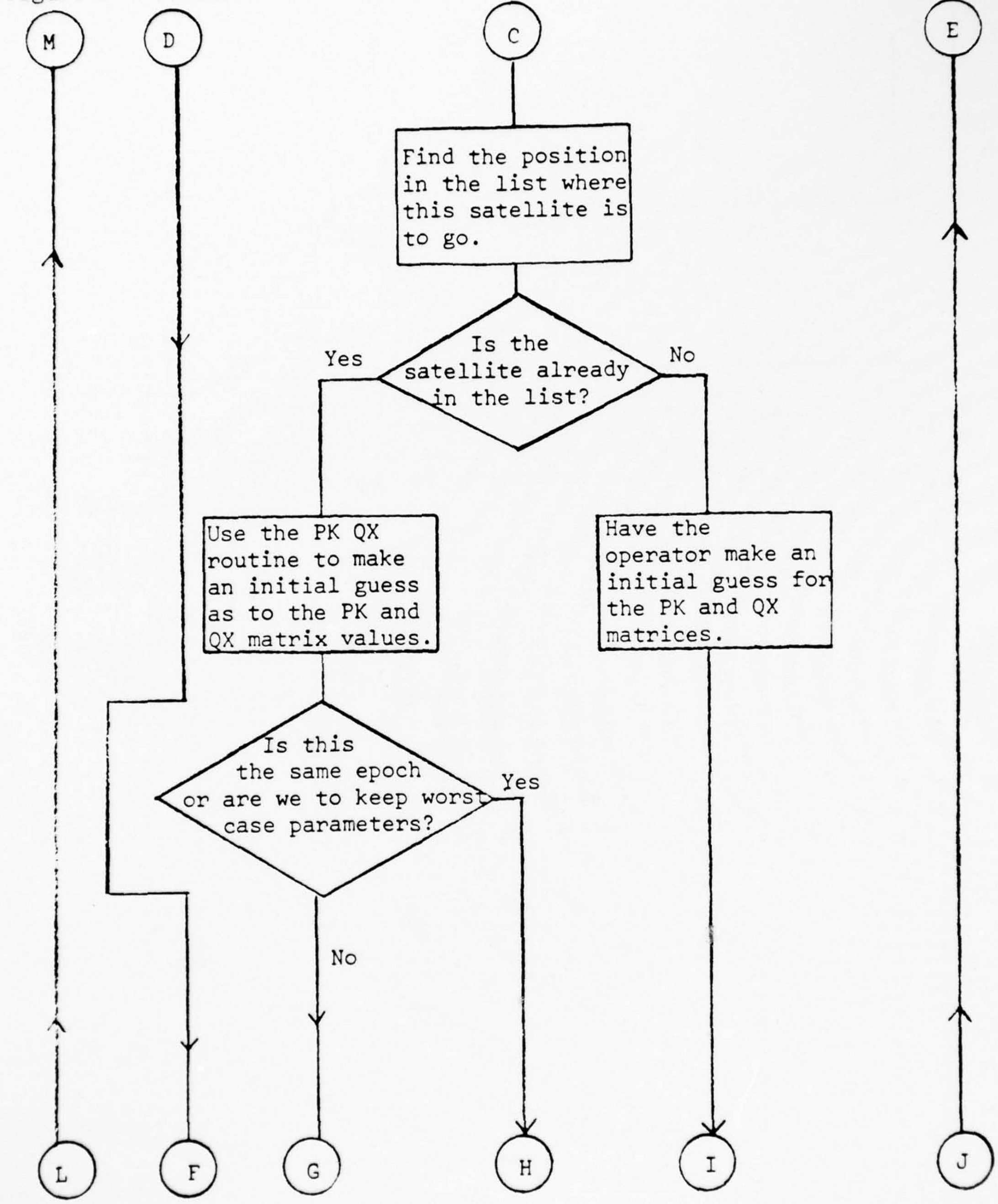


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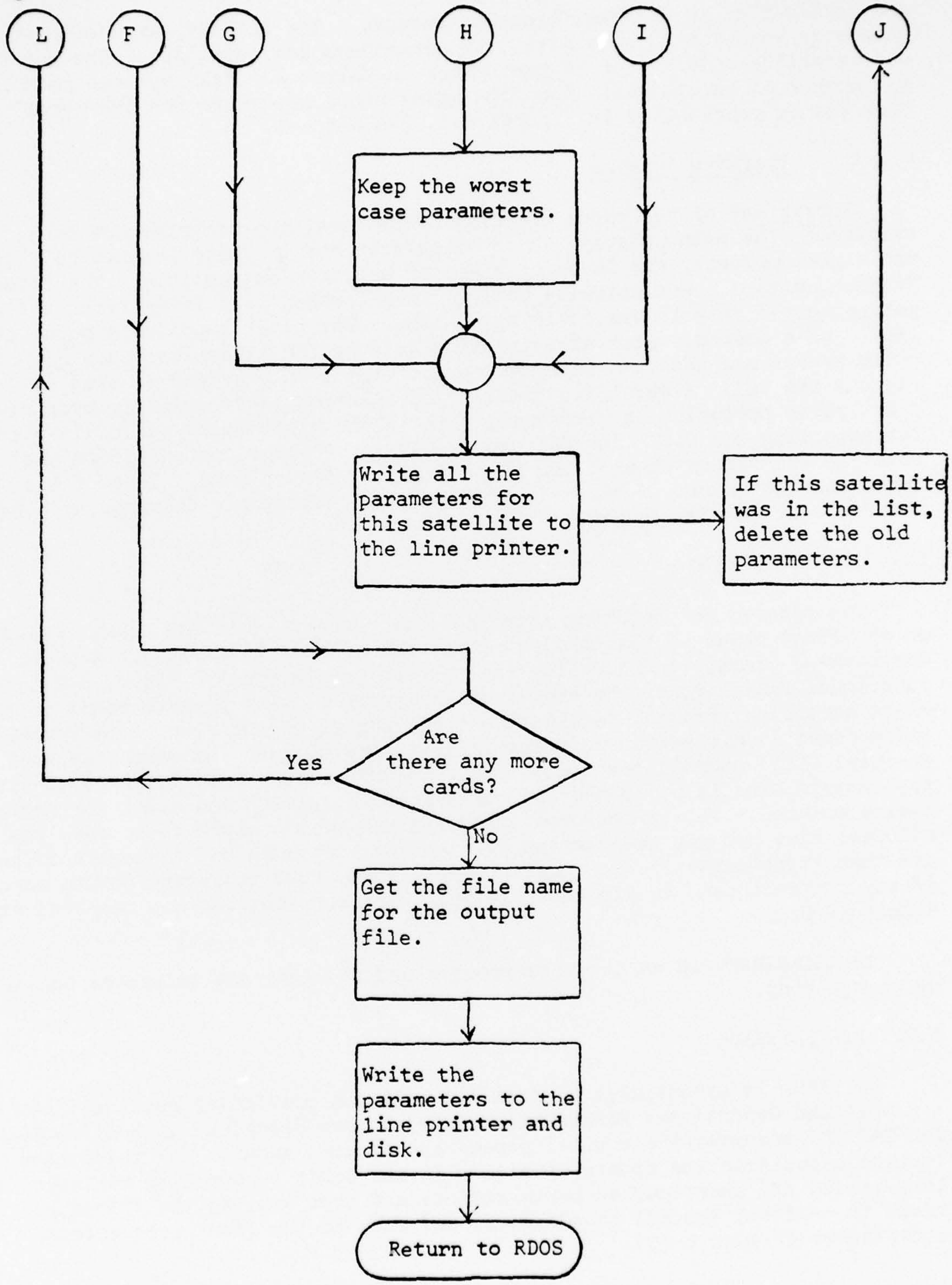
Figure 6-4 (Continued)



and declination in degrees, minutes, seconds. The SATLIST can accomodate parameter sets for 50 satellites, 12 parameters per satellite. The set for one satellite consists of 24 parameters in Kaltrack. Kaltrack can read in any number of satellites. The file maintenance procedure for the three programs is represented in Figures 6-5, 6-6 and 6-7.

6.2.3. Tracking Systems

During any of the three tracking operations, two programs are being executed. The main program, which calculates the tracking parameters, has the higher priority and is being executed as often as possible. The second program, with a lower priority rating, is a Command Line Interpreter which gets executed only in available spare time. The clock supplies a pulse every $1/60^{th}$ of a second. Each star-tracking calculation takes less than one clock pulse period and the time left over out of the $1/60^{th}$ second is used to execute the CLI. A satellite-tracking calculation takes slightly over five clock pulse periods. The remaining time in the sixth pulse period is used for executing the CLI. The CLI can accept four-letter commands from the teletype and set or clear flags accordingly. (Figure 6-8). These flags determine the course of the main program. All acceptable commands have been described in the User's Manual.

6.2.3.1. STARTRACK

The program for tracking stars has been described in PAR Report #75-32 on the first phase of this project (Contract #F30602-75-C-0144). A short description is repeated here for the sake of completeness. Figure 6-9 is a functional flowchart of the star-tracking program. The program reads in the right ascension (RA) and declination (DEC) angles of the star to be tracked and defines a unit vector pointing in that direction in the earth-centered inertial (ECI) coordinates. Coordinate transformation from ECI axes to site topocentric axes is performed and corresponding azimuth and elevation angles are calculated. This transformation makes use of the clock to compute the sidereal time and get the current site vector. Azimuth and elevation offsets are then transferred to the mount servos. Then, RDOS tasks the CLI to check if any commands came in via the teletype. These operations are repeated at almost 60 Hz.

The STARTRACK is an accurate program and it keeps the telescope locked on to the star.

6.2.3.2. SATTRACK

SATTRACK is essentially a satellite position prediction routine based on a Simplified Generalized Perturbation (SGP) theory described in Section 3. The SATLIST provides the orbital parameters at some epoch. The prediction routine calculates the updated position in the orbit by applying the secular, long-period and short-period perturbations and then applies the transformations to -- first the ECI coordinates, and then to the local topocentric coordinates (Figure 6-10).

Figure 6-5 File Maintenance for STARTRACK Routine "STARLIST"

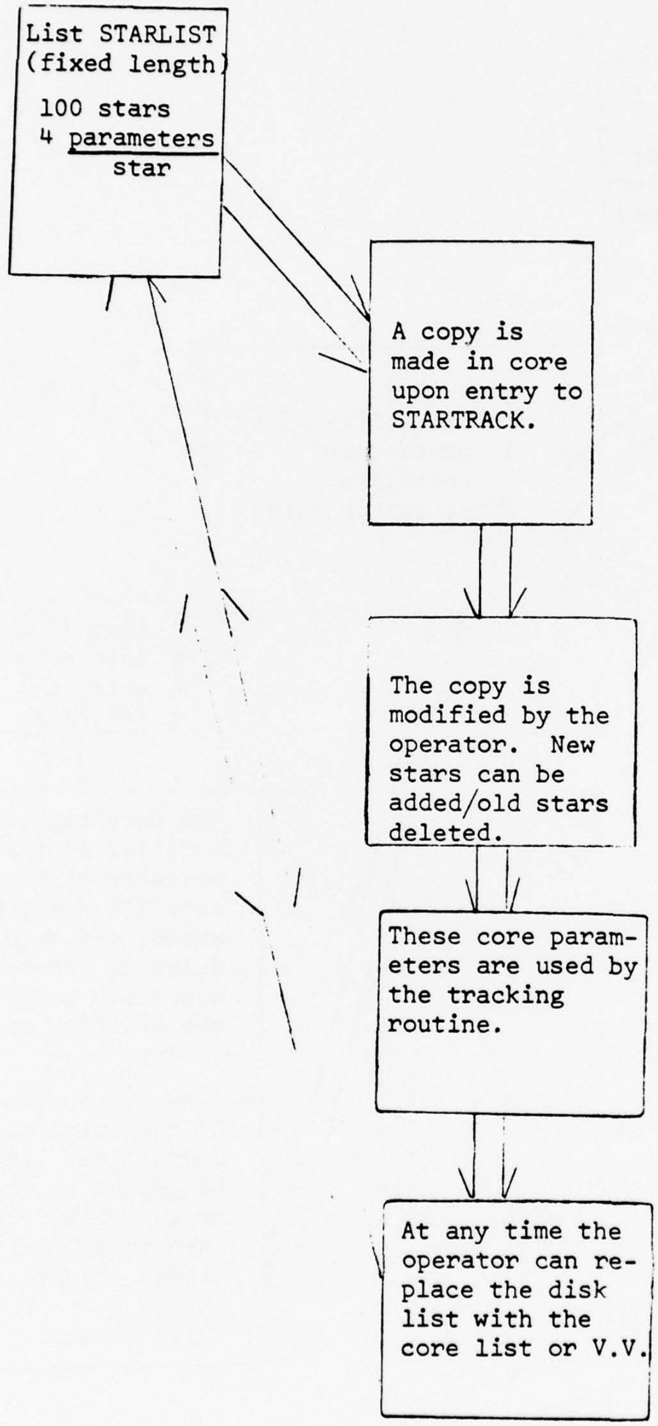


Figure 6-6 File Maintenance for SATTRACK Routine "SATLIST"

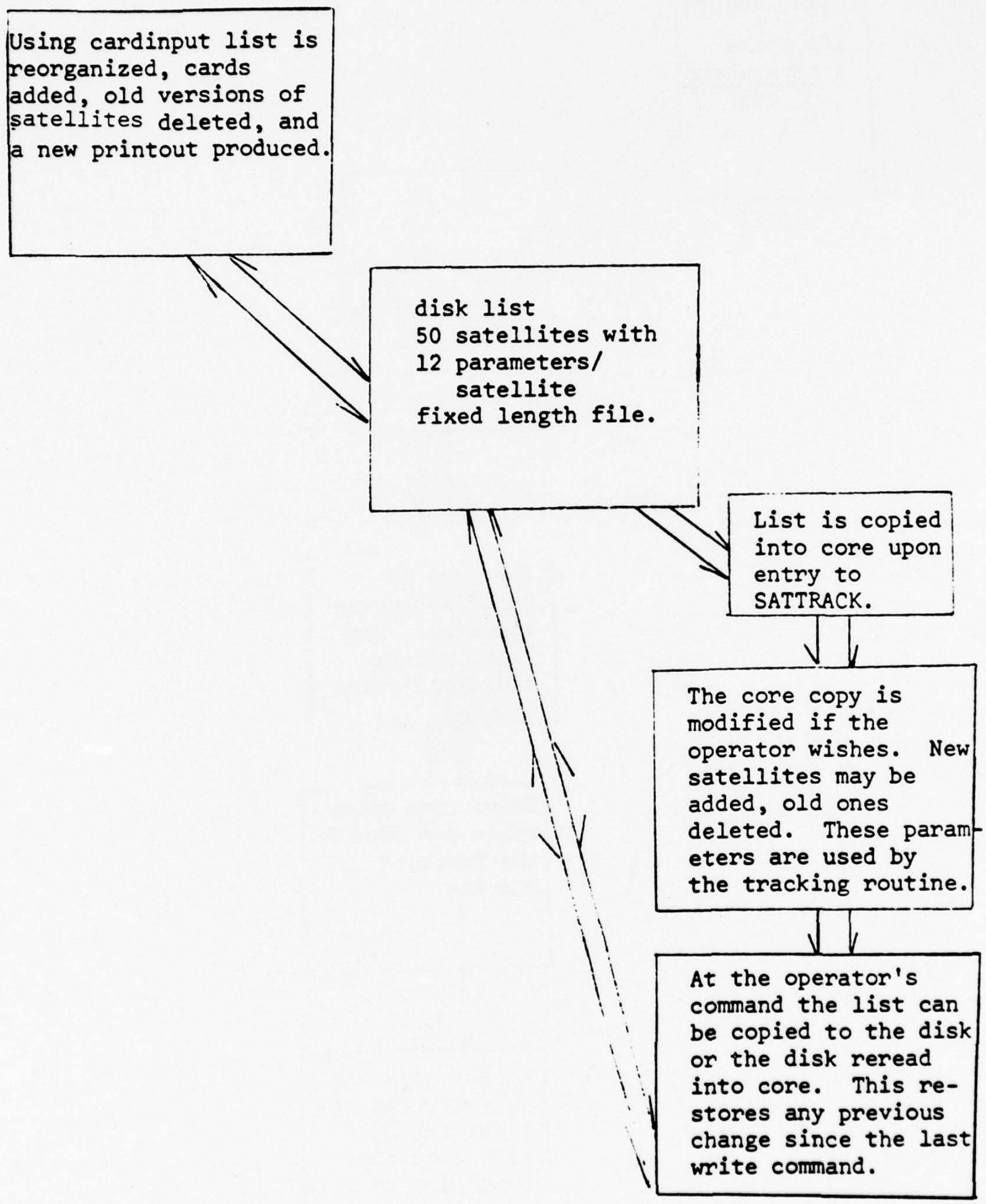


Figure 6-7 File Maintenance for Kalman Filter Routine "KALLIST"

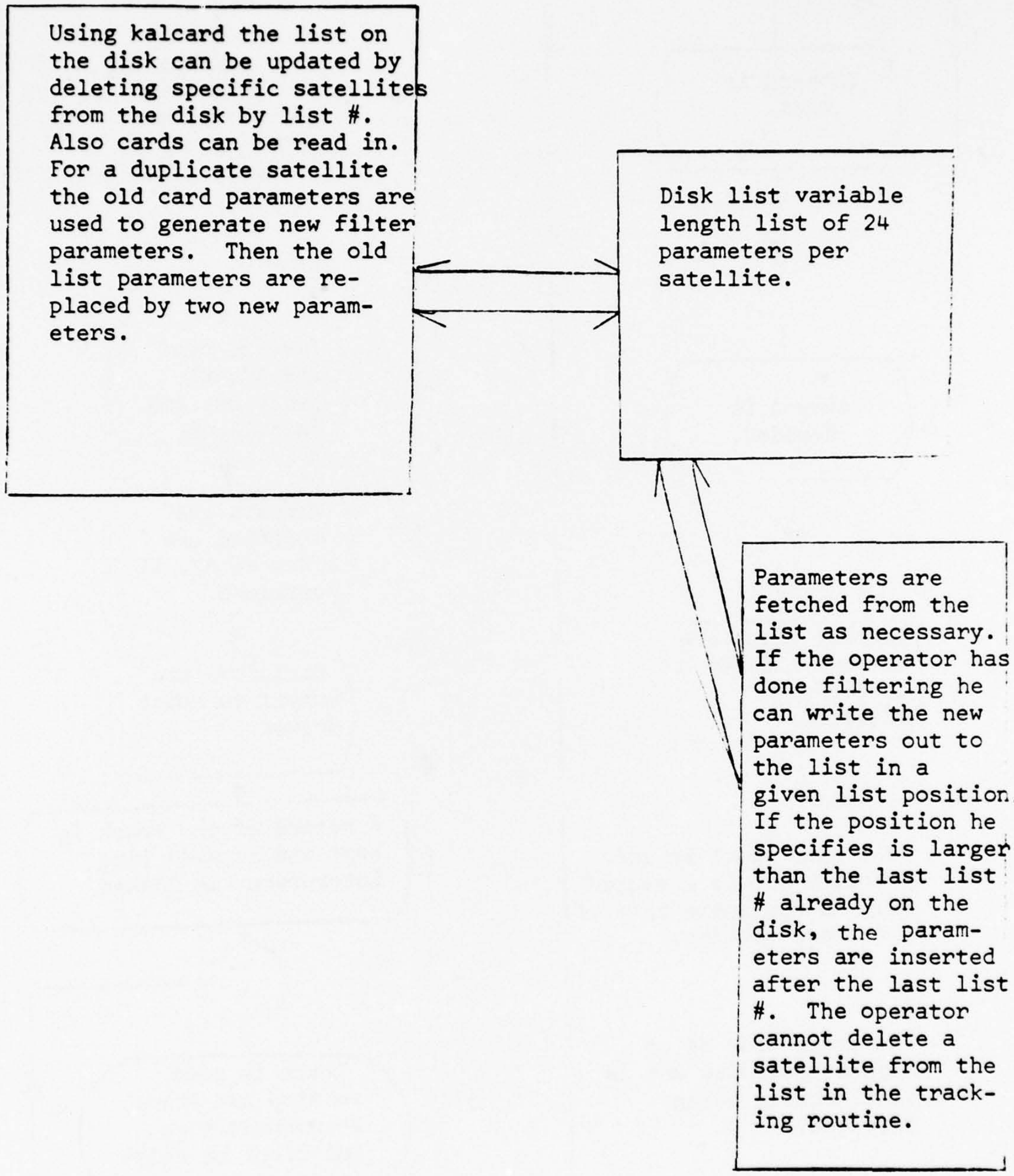
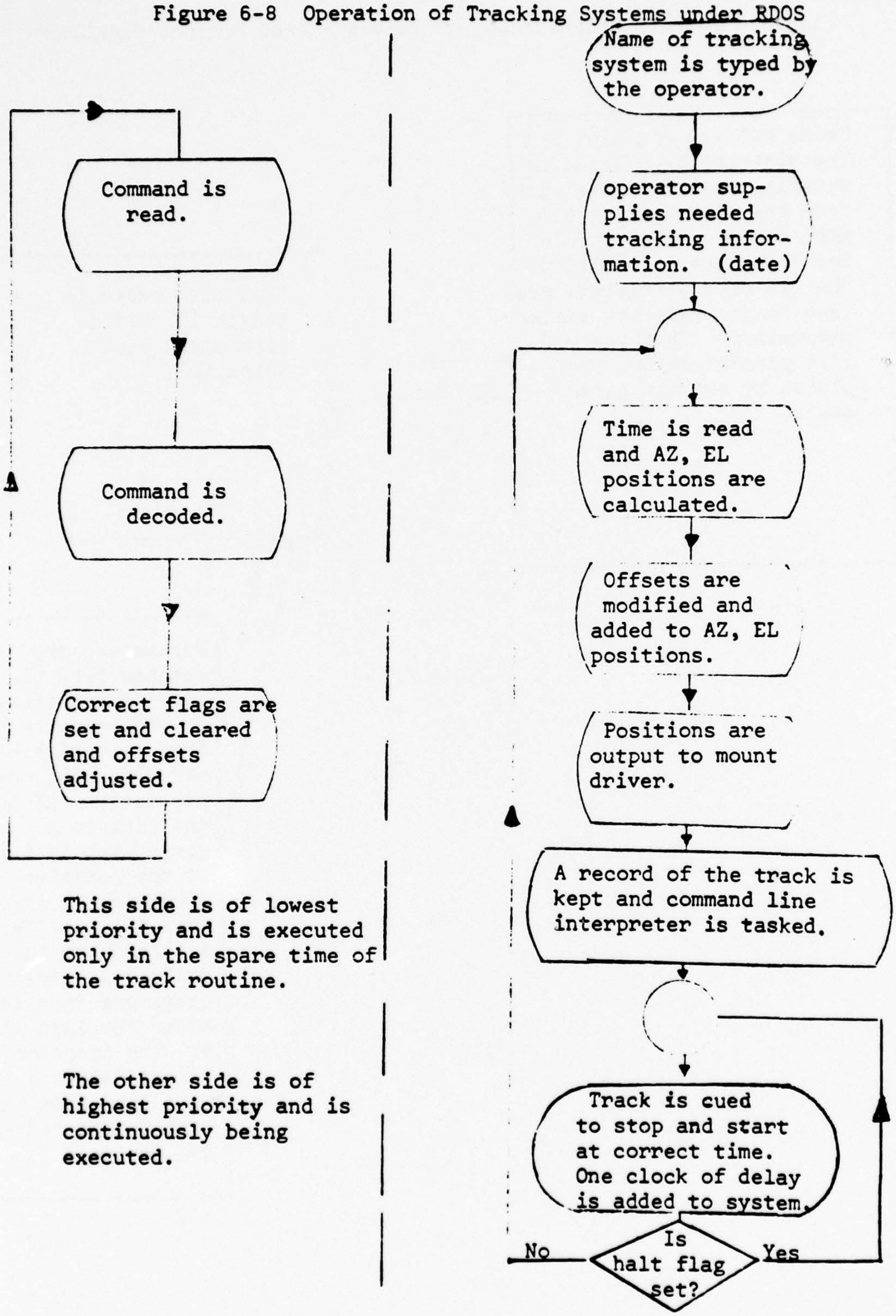


Figure 6-8 Operation of Tracking Systems under RDOS



TASK 1

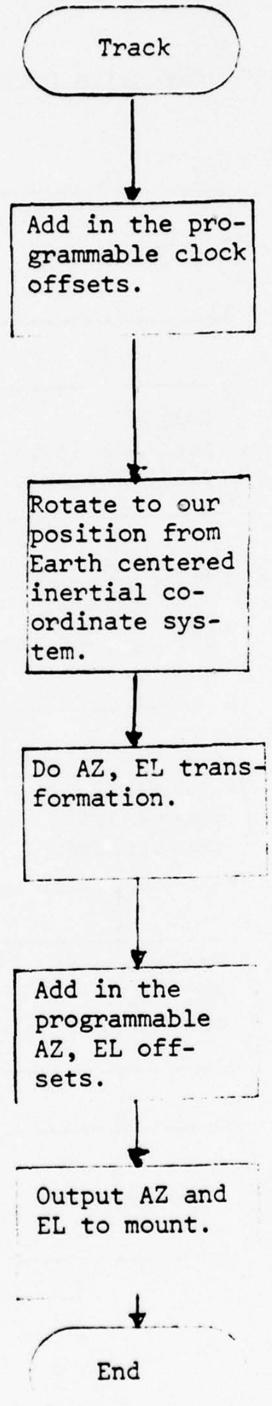
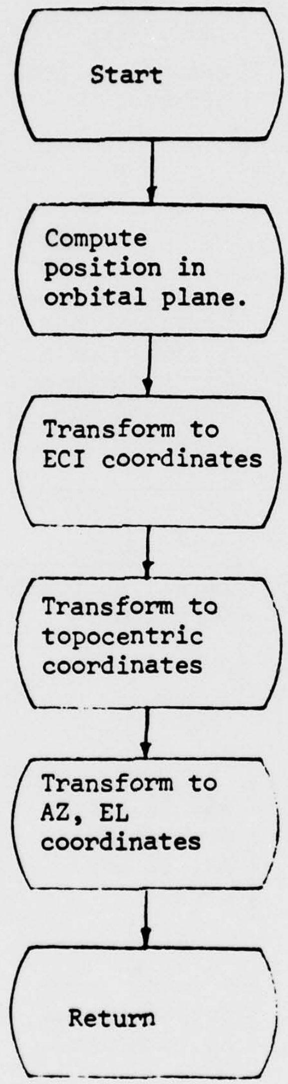


Figure 6-9 Flowchart for Task 1

Figure 6-10 Satellite Prediction Routine



6.2.3.3. KALTRACK

When it is possible to measure the actual azimuth and elevation of the satellite as a function of time, KALTRACK can be used to track the satellite and update the orbital parameter, simultaneously. A NOVA-TV interface has been designed and fabricated for the specific purpose of measuring the actual satellite differential position after it is acquired in the field of view. This hardware is described in Section 7 of this report.

According to the theory described in Section 5, the satellite parameters may be updated using the measured azimuth-elevation angles of the satellite using the Kalman filter technique (Figure 6-11).

6.3. HARDWARE

6.3.1. Computer-Mount Interface

Only the computer-mount interface designed by PAR will be described in short for the sake of completeness. A detailed design description was submitted as an Interim Report. Figure 6-12 schematically shows the hardware-software interface chart. Supplying the new look angles to the mount, and reading the current mount position, are the two functions performed through the interface. Error codes are also generated which indicate whether or not the mount is slewing or at an endstop.

Figure 6-13 is a schematic of the data link between the CPU and the mount for relaying the new look angles to the mount. Figure 6-14 shows the mount to CPU interface for reading the current mount position and 6-15 the clock CPU interface.

6.3.2. Computer-TV Interface

A computer-TV interface was fabricated to measure the actual satellite position. When the satellite is acquired on the TV screen, cross-hairs on the screen can be moved to its observed position, and a push button switch supplies these azimuth-elevation offsets to the Kalman filter as inputs. The filter applies a calculated correction. This procedure can be repeated by moving the cross-hairs to the new satellite position. The hardware and the operation has been described in Section 7.

6.4. SUBROUTINE AND PROGRAM FUNCTION AND INDEX

6.4.1. STARTRACK

The routines described in this subsection are exclusive to the STARTRACK program. STARTRACK is the main driver which calls the appropriate routine to implement the track.

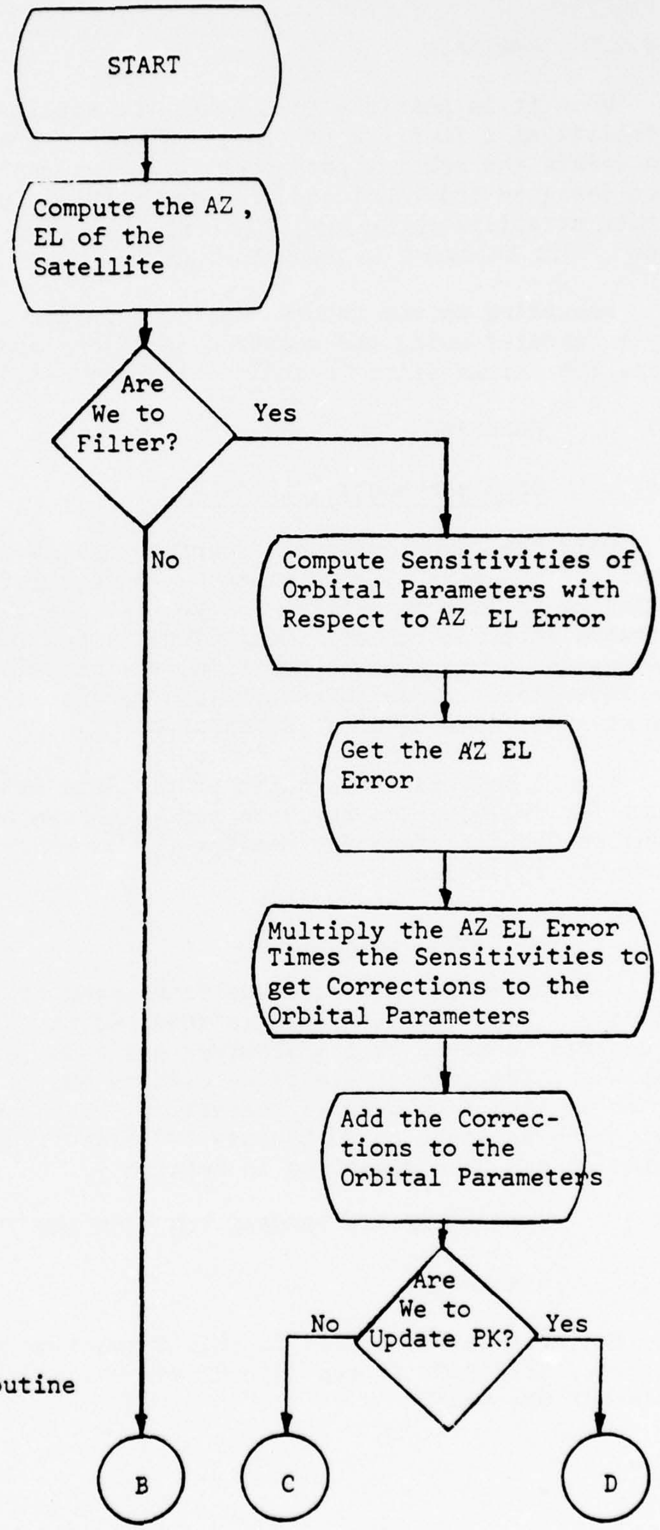
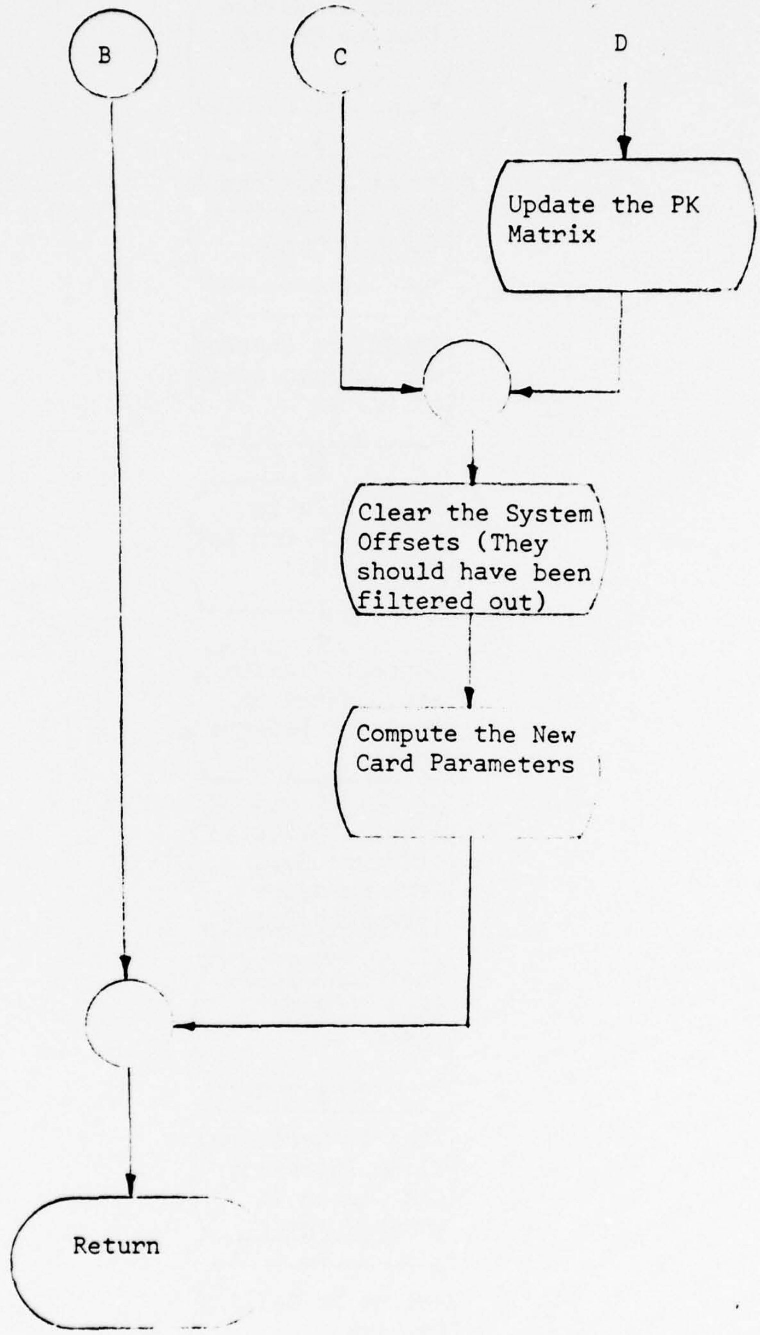


Figure 6-11 Kalman Filter Routine

Figure 6-11 (Continued)



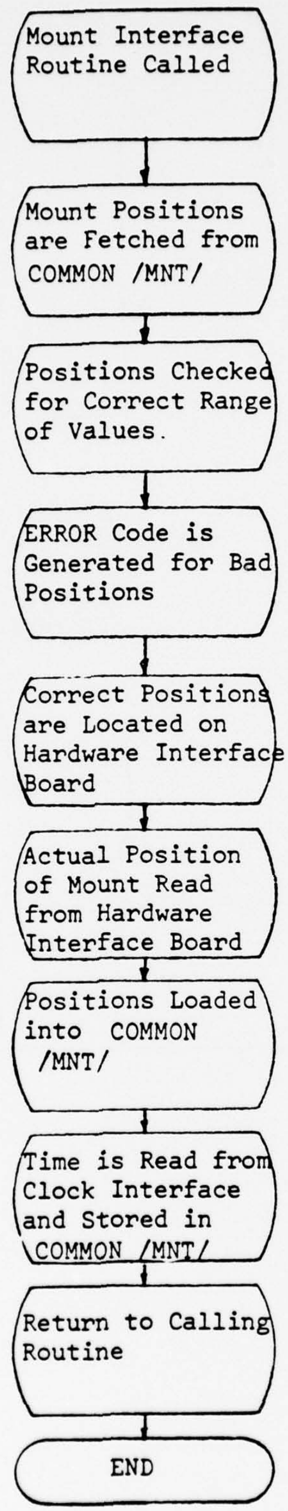


Figure 6-12 Hardware Software Interface Flowchart

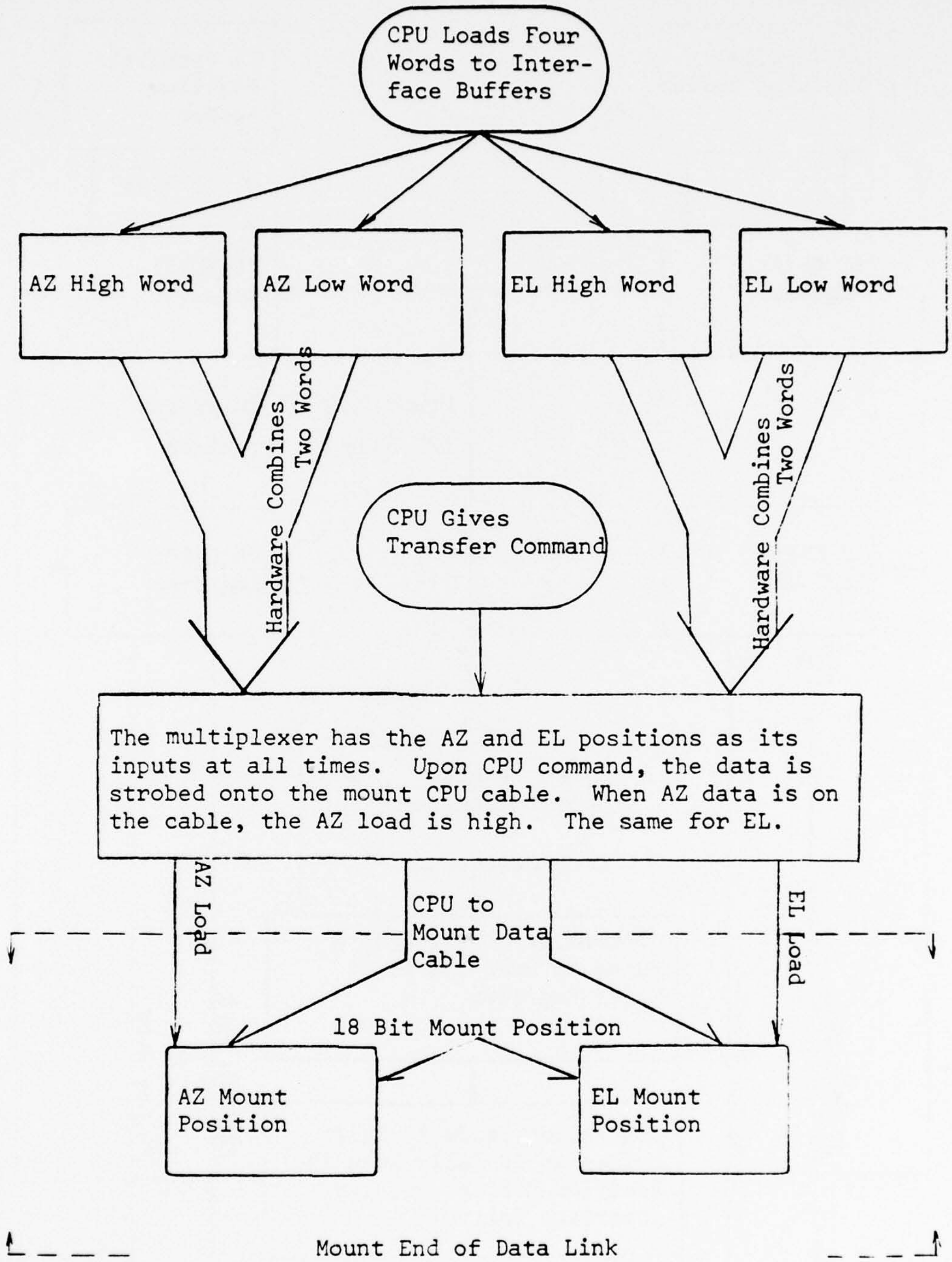
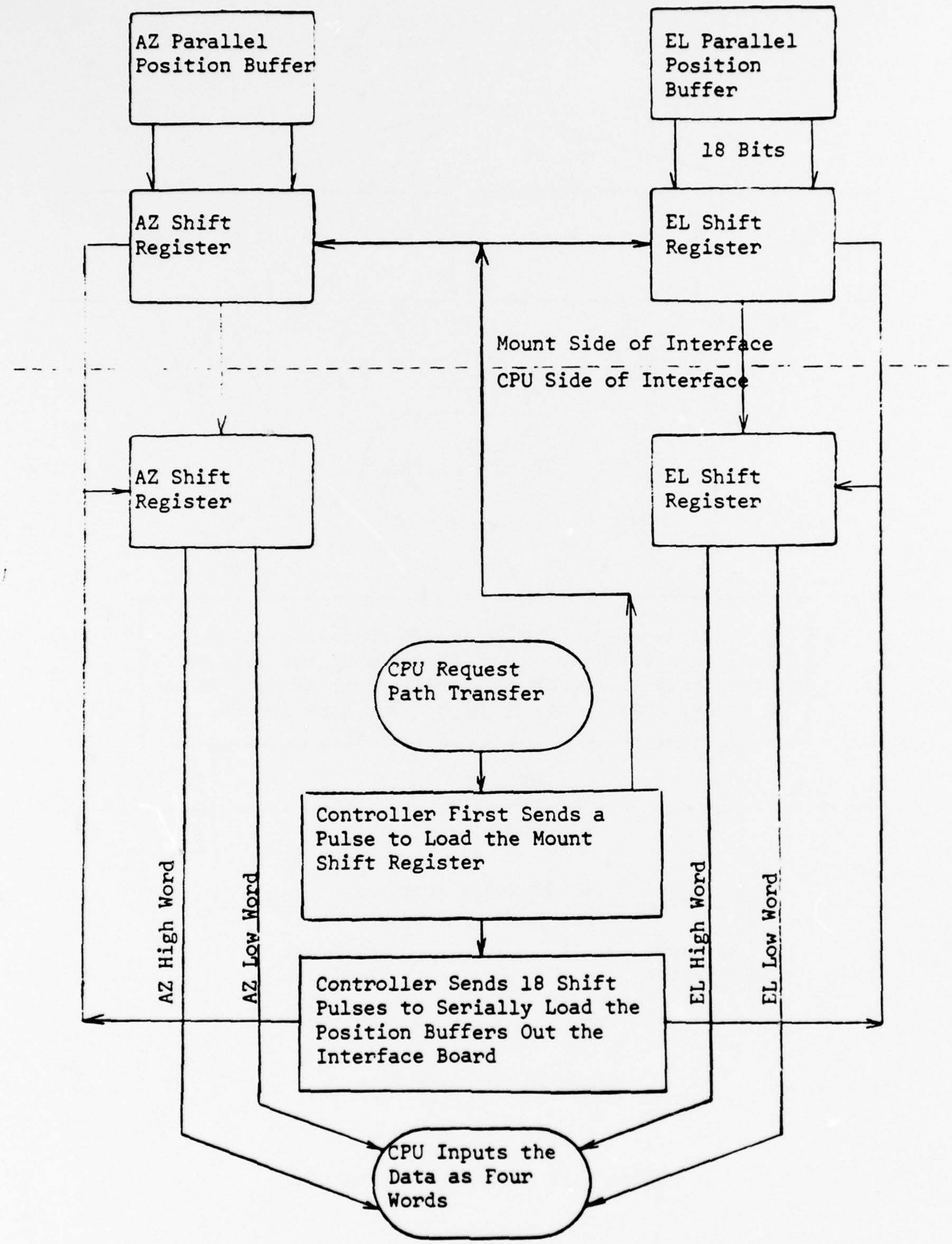


Figure 6-13 CPU End of Data Link

Figure 6-14 Mount to CPU Interface to Input Mount Position



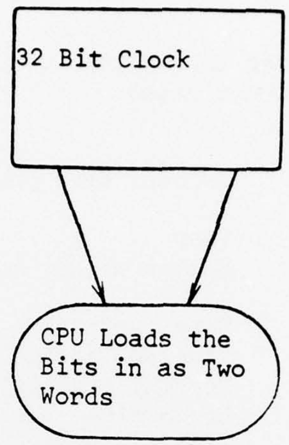


Figure 6-15 Clock CPU Interface

6.4.1.1. COMND1

This is the command line interpreter for the STARTRACK program which has no real input or output, but serves to modify parameters and flags of other routines, according to the operator's commands.

6.4.1.2. PAR3

This routine initializes the stellar tracking routine by supplying output parameters in the common areas.

COMMON/SDRIVE/ #	
[RX, RY, RZ]	Initial star position
/I1/	
IYR	Year
IDAY	Number of day in the year
/I2/	
LAT	Site latitude
COSLAT, SINLAT	Latitude cosine and sine functions
STIME	Current sidereal time
OSTIME	Epoch time
OTSINCE	Time since epoch
SINS, COSS	Sine and Cosine of the sidereal time
VXY, VZ	Related to flatness of earth

6.4.2. SATTRACK4

This routine links all the appropriate routines to do a passive satellite track. All the subroutines exclusive to this system have been described in this subsection.

6.4.2.1. COMND

This is the SATTRACK analog of COMND1 and is the command line interpreter for the passive satellite tracker routine. It has no real inputs or outputs except that it modifies parameters and flags in response to operator commands to control the course of other subroutines.

6.4.2.2. ORBIT

This routine performs the transformation of parameters from the satellite orbital coordinate system to the earth-centered inertial system (ECI) parameters.

Input Parameters

TRUEU	A dummy variable (true anomaly)
/05/	
RMAG	Magnitude of satellite vector in ECI axes

The name within the slashes /SDRIVE/ is the name of the common block.

SNODE, CNODE	Sine and cosine of (RA of the ascending node)
SINI, COSI	Sine and cosine of i (inclination)
RMGDT	Magnitude of rate of change of RMAG
RVDT	Radial component of velocity
/I2/	
COSLAT	COS (Latitude)
SINS, COSS	Sine and cosine of the sidereal time
CVXY, CVZ	Constants due to oblateness of earth
Output Parameters	
/SDRIVE/	
RX, RY, RZ	X,Y, Z in the ECI system

6.4.2.3. PAR5

This routine performs a similar function in SATTRACK as does PAR3 in STARTRACK. It has some I/O with the operator which involves initializing the program as described in the User's Manual under 2.2.2.4.

6.4.2.4. SEMI (Eccentricity, Mean Motion, Inclination)

This function computes the semi-major axis of the orbit given the eccentricity, mean motion and the inclination. It is also used in KALTRACK.

Input Parameters as arguments are the dummy variables representing eccentricity, mean motion and inclination as shown above.

Output Function

SEMI	Semi-major axis

6.4.2.5. TRANS (AZ, EL)

This routine serves to perform the transformation from the ECI to site topocentric axes and then to calculate the azimuth and elevation angles.

Input Paramters

/SDRIVE/	
RX, RY, RZ	Position of object in ECI coordinate system
/I2/	
COSLAT, SINLAT	Sine and cosine of latitude
SINS, COSS	Sine and cosine of sidereal time
CVXY, CVZ	Constant for flatness of earth
LAT	Latitude

Output Parameters

AZ, EL	AZ, EL positions of objects

6.4.2.6. VARI (TRUE, TSINCE)

Using the classical orbital parameters and the time since epoch (TSINCE), this routine calculates the satellite position in the orbital coordinate system. Common area /I1/ is the input to this routine and the output parameters are loaded into the common area /01/, /02/, /05/, /TEST/

/I1/	
EO, EPOCH, IO, MO, NDOTO, NDOT6, NO, NODEO, OMEGO, REVO	
/SDRIVE/	
RX, RY, RZ	
/01/ IFLG AM, EM, IM, NODEM, OMEGM, NDOTM, REVM, LM, NM	Mean Values of parameters
/02/ ELONG, LLONG, EXLNG, OMEGL TEMP, TEMPL	Long-period perturbed values
/TEST/	
IS, NODES	Short-period perturbed values
/05/	
RMAG, SNODE, CNODE, SINI, COSI	
RMGDT, RVDT	

6.4.2.7. CARD

This routine reads the NORAD two-card data set into the core and into the following common areas and sets error flag (FLAGE) if there is an error.

/I1/	EPOCH, YR, REVO, NO, IO, EO, MO, NODEO, OMEGO, NDOTO, NDOT6
/CARDC/	SATNO, FLAGE

6.4.2.8. CARDINPUT

This is a main program which updates the satellite list by introducing the satellite data read from the cards in their proper order in the SATLIST. If the data of any satellite being read in already exists in SATLIST, then the old data is overwritten with the new parameter set (2.2.2.2.). The variables used are all those listed under CARD.

6.4.3. <u>KALTRACK</u>

This program uses Linearized Kalman Filter to perform an active satellite track, with the operator supplying azimuth and elevation angle measurements of the observed satellite. The observations are used to form an error signal which is used to apply a correction to the orbital elements. When there are no observations forthcoming, KALTRACK uses a variation of SATTRACK to continue tracking the satellite by prediction alone. KALTRACK is the driver program to perform the tracking function and has no inputs and outputs of its own. Most variables occur in this program.

6.4.3.1. EXANM (Mean Anomaly, Eccentricity)

This function is used in both KALTRACK and SATTRACK4 to calculate the eccentric anomaly using the eccentricity and mean anomaly. Kepler's equation is solved by employing a simple iteration technique.

Input Parameters are the arguments Mean Anomaly and Eccentricity as shown above.

Output Function

EXANM	Eccentric anomaly

6.4.3.2. KAL (TSINCE, AØ, EØ)

This is the main Kalman filter routine which updates the orbital parameters based on the azimuth and elevation measurements and transmits the predicted az-el angles to the mount for tracking purposes. It detects any mount or time offsets from the system.

Input Parameters

TSINCE		
/I1/	EPOCH, YR, REVO, NO, IO, EO, MO, NODEO, OMEGO, NDOTO, NDOT6, IYR, DAY	Card parameters
/I2/	COSLAT, SINLAT, STIME, OTSINCE, VXY, VZ SINS, COSS, LAT	Earth's position
/KALF/	XHATK, PK, QX, CRK1, NCDOT, NCK, DELTK, KALFLAG	

Output Parameters

/I1/	EPOCH, YR, NO, IO, EO, MO NODEO, OMEGO	Card parameters
/OFSET/	OAZ, OEL, OT System offset	
AØ, EØ	AZ, EL Look angles	
/01/	EM	

6.4.3.3. KALCOMND

This routine is the command line interpreter for KALTRACK. It has no real inputs or outputs, but it modifies the parameters and flags of other routines.

6.4.3.4. KALPAR

This routine intializes the Kalman filter tracking system with the following output parameter.

/I1/	EPOCH, YR, REVO, NO, IO, EO, MO, NODEO, OMEGO, NDOTO, NDOT6, IYR, DAY
/I2/	COSLAT, SINLAT, STIME, LAT, OSTIME, OTSINCE, VXY, VZ, SINS, COSS

6.4.3.5. KALP (TRUEU, TSINCE, XBARK, ROWTK1, AZØ, ELØ)

This routine performs the prediction part of the filter. Using the most recent orbital parameter estimate it predicts the state transition matrix and the az-el angles.

Input Parameters

TSINCE	TIME science epoch	
/I1/	EPOCH, YR, REVO, NO, IO, EO, MO, NODEO, OMEGO, NDOTO, NDOT6	Card parameters
/I2/	COSLAT, SINLAT, LAT, CVXY, CVZ, SINS, COSS	Earth's position

Output Variables

/01/	OMEGM, NODEM, NM, EM, REVM, IFLG, IM, LM, NDOTM
/02/	OMEGL, TEMP, TEMPL, ELONG, EXLNG, LLONG
/05/	RVDT, CNODE, COSI, SINI, RMAG, RMGDT
/TEST/	IS, NODES
/SDRIVE/	RX, RY, RZ

TRUEU, XBARK(6), ROWTK1, AZØ, ELØ

6.4.3.6. KALCARD

This is a main program which updates the KALLIST according to the new card parameter set fed in.

6.4.3.7. OBSER (AZ, EL)

This routine passes the observed angles to KAL.

6.4.3.8. PKQX (CARD1, CARD2, PK, QX)

This routine supplies the initial values for the initial error covariance PK and the system noise covariance QX based on the new and old card parameters.

Input Parameters

CARD1	Old set of card parameters
CARD2	New set of card parameters

Output Parameters

PK	Error covariance of orbital parameters
QX	System noise covariance matrix

6.4.4. <u>General Purpose and Common Routines</u>

This subsection describes the routines which are not exclusive to any one of the above three systems and perform frequently required general-purpose functions.

6.4.4.1. Matrix Operations

6.4.4.1.1. ABAT (A, B, C, I, J, D)

This provides the following matrix operation

$$C = ABA^T$$

where A and B are (IxJ) and (JxJ) input matrices and C is the (IxI) output matrix. D is an (IxJ) matrix used for intermediate result.

6.4.4.1.2. ABT (A, B, C, I, J, K)

This performs the matrix operation

$$C = AB^T$$

where A and B are (IxJ) and (JxK) input matrices and C is the (IxK) output matrix.

6.4.4.1.3. MATMUL (A, B, C, I, J, K)

As the name implies, this routine performs the matrix product, C=AB where C, A and B are (I,K), (I,J) and (J,K) matrices respectively.

6.4.4.1.4. MATSUB (A, B, C, I, J)

This routine outputs the difference of two (I,J) matrices A and B as C.

6.4.4.1.5. MATADD (A, B, C, I, J)

This routine outputs the sum of two matrices A and B as C, all being of dimensions (I,J).

6.4.4.2. TRIGONOMETRIC OPERATIONS

6.4.4.2.1. MOD2P (X)

This routine takes the MOD(2π) of a number X.

6.4.4.2.2. AKTAN (XX, YY)

This function takes the arc tangent when the x and y components are supplied to it as dummy variables. The output angle is between 0 and 2π.

6.4.4.3. CLOCK ROUTINES

All routines having anything to do with time have been categorized under this heading.

6.4.4.3.1. TIME (TSINCE)

This routine calculates the time since the last update of the orbital parameters and the sidereal time using the current time of day and values of those parameters at the beginning of the day.

Input Parameters

/OFSET/	OT (System time offset)
/I2/	OSTIME, OTSINCE (Sidereal time and tsince at 24:00 the day before.)

Output Parameters

TSINCE	Time since epoch
/I2/	STIME, SINS, COSS (Sidereal time and its sine and cosine.)

6.4.4.3.2. CLCK (MS)

CLCK returns the time of day in milliseconds.

6.4.4.3.3. CLCK1 (WORD1, WORD2)

CLCK1 returns the time of day as two words: i.e., tens of seconds and milliseconds. Output: first two locations on the stack.

6.4.4.4. SYSTEM ROUTINES

6.4.4.4.1. FDELY (X)

This is an RDOS routine which can be employed to delay the advance of the program by a desired number of clock pulses.

6.4.4.4.2. FTASK (SUBROUTINE_NAME, $ERROR_RETURN, PRIORITY)

This is another RDOS routine which sets up a task control block for a new task.

6.4.4.5. MOUNT-COMPUTER INTERFACE ROUTINES

6.4.4.5.1. MNTS

This routine starts the mount-computer communications by synchronizing the two devices.

6.4.4.5.2. MOUNT (ERR)

This routine converts the input angle from radians to revolutions and outputs the result to the assembly language driver routine.

Input Parameters

/MNT/ AZØ, ELØ, (AZ, EL look angles)

Output Parameters

/MNT/ TIME2, AZ1, EL1 (AZ, EL position and time in days of position read)

6.4.4.5.3. MNT1 (AZ, EL)

This is an assembly language driver to output the angles to the mount. Input parameters are the first two locations on the stack.

6.4.4.5.4. MNT2 (AZ, EL, AZER, ELER, FLAG2, FLAG3)

This also is an assembly language driver which accepts input from the mount in the first six locations on the stack.

Output Parameter

AZ, EL	AZ, EL position in revolutions
AZER, ELER	AZ, EL hardware error positions
FLAG2, FLAG3	Video interlace control bits

6.4.4.5.5. ERROR (ERR)

(ERR) Input error code

This routine decodes any errors incurred during the positioning of the mount.

6.4.4.6. SEARCH (Input Azimuth, Input Elevation, Output Azimuth, Output Elevation, Mode Flag)

This routine implements a spiral search in the line of sight coordinate system.

MODE = 0 if search calculation is to be done
= 1 is search is to be setup & controlled
= 2 if search is to be controlled

Input Parameters

/SRCH/ & /OFSET/ OAZ, OEL, TIME

Output Parameters

/OFSET/ OAZ, OEL system AZ, EL offsets

6.4.5. Alphabetical List of Subroutines

Number	Name	Section Number	Page Number
1	ABAT	6.4.4.1.1.	6-31
2	ABT	6.4.4.1.2.	6-31
3	AKTAN	6.4.4.2.2.	6-31
4	CARD	6.4.2.7.	6-28
5	CARDINPUT	6.4.2.8.	6-28
6	CLCK	6.4.4.3.2.	6-32
7	CLCK1	6.4.4.3.3.	6-32
8	COMND	6.4.2.1.	6-26
9	COMND1	6.4.1.1.	6-26
10	ERROR	6.4.4.5.5.	6-33
11	EXANM	6.4.3.1.	6-28
12	FDELY	6.4.4.4.1.	6-32
13	FTASK	6.4.4.4.2.	6-32
14	KAL	6.4.3.2.	6-29
15	KALCARD	6.4.3.6.	6-30
16	KALCOMND	6.4.3.3.	6-29
17	KALP	6.4.3.5.	6-30
18	KALPAR	6.4.3.4.	6-29
19	KALTRACK	6.4.3.	6-28
20	MATADD	6.4.4.1.5.	6-31
21	MATMUL	6.4.4.1.3.	6-31
22	MATSUB	6.4.4.1.4.	6-31
23	MOD2P	6.4.4.2.1.	6-31
24	MOUNT	6.4.4.5.2.	6-32
25	MNTS	6.4.4.5.1.	6-32
26	MNT1	6.4.4.5.3.	6-33
27	MNT2	6.4.4.5.4.	6-33
28	OBSER	6.4.3.7.	6-30
29	ORBIT	6.4.2.2.	6-26
30	PAR3	6.4.1.2.	6-26
31	PAR5	6.4.2.3.	6-27
32	PKQX	6.4.3.8.	6-30
33	SATTRACK4	6.4.2.	6-26
34	SEARCH	6.4.4.6.	6-33
35	SEMI	6.4.2.4.	6-27
36	STARTRACK	6.4.1.	6-19
37	TIME	6.4.4.3.1.	6-32
38	TRANS	6.4.2.5.	6-27
39	VARI	6.4.2.6.	6-27

6.4.6. Alphabetical List of I/O Variables

	Math Equivalent	Occurs in Program
A	a - semi major axis	KALP, VARI
AM	a-mean	KALP, VARI
AO	a-original	KALP, VARI, KAL
AP	AZ predicted angle	SEARCH

AØ	Azimuth pointing angle without any offsets	KAL
AZ	Azimuth	Most programs
AM	Mean value of semi-major axis	VARI, KALP
AZØ	Azimuth output angle	KALTRACK, STARTRACK, MOUNT, SATTRACK, KALP
AZ1	Azimuth input angle	KALTRACK, STARTRACK, MOUNT, SATTRACK
AZER	Azimuth error from hardware interface	MNT2
CARD1	First set of card parameters	PKQX
CARD2	Second set of card parameters	PKQX
CNODE	Cos (node)	KALP, ORBIT, VARI
COSI	Cos (inclination)	KALP, ORBIT, VARI
COSLAT	Cos (latitude)	KAL, TRANS, KALP, ORBIT PAR3, PAR5, KALPAR
COSS	Cos (Sidereal time)	TIME, TRANS, KALP, ORBIT PAR3, PAR5, KALPAR
CRK1	Covariance of AZ, EL inputs	KAL
CVXY	Constant due to oblateness of earth	KALP, TRANS, ORBIT
CVZ	Constant due to oblateness of earth	KALP, TRANS, ORBIT
DAY (or IDAY)	Day of year	KALPAR, PAR3, PAR5, KALPAR Tracking routines, KAL
DELTK	Time since last Kalman filter	KAL
EØ	Elevation pointing angle without any offsets	KAL
EL	Elevation	Most programs
ELØ	Elevation output angle	Main tracking routine and MOUNT, KALP
ELER	Elevation error from hardware interface	MNT2
ELONG	Long periodic term of eccentricity	KALP, VARI
EL1	Elevation input angle	Main Tracking routines and MOUNT
EM	Mean eccentric anomaly	KALP, KAL, VARI
EO	Original Eccentricity	KALPAR, KALP, KAL, VARI, CARD, CARDINPUT
EPOCH	Time of original values	KALPAR, CARD, KALP, KAL, VARI, CARDINPUT
ERR	Error code for mount positioning	MOUNT, ERROR
EXLNG	Long periodic term for eccentric anomaly	KALP, VARI
FLAG2, FLAG3	Flags from hardware interface	MNT2
FLAGE	Error code for subroutine "CARD"	CARD, KALCARD, CARDINPUT
IDAY	Same as DAY	
IFLG	Used to short cycle VARI	VARI, KALP

IM	Mean inclination	VARI, KALP
IO	Original inclination	KAL, VARI, KALP, CARD, CARDINPUT, KALPAR
IS	Short periodic term for inclination	VARI, KALP
IYR (or YR)	Year	CARDINPUT, CARD, main tracking routines, PAR3, PAR5, KALPAR
KALFLAG	Flag to start Kalman filter	KAL
LAT	Latitude	KALP, KALPAR, TRANS, KAL, PAR3, PAR5, KALPAR
LLONG	Orbital longitude	VARI, KALP
LM	Mean orbital longitude	VARI, KALP
MO	Mean anomaly	KALPAR, VARI, KALP, CARD, CARDINPUT, KAL
MODE	Mode flag for search routine	SEARCH
NCDOT	Corrected (estimated) NDOTO	KAL
NCK	Unused variable	KAL
NDOTM	Mean value of NDOTO	VARI, KALP
NDOTO	First derivative of mean motion	VARI, KALP, CARD, CARDINPUT, KAL, KALPAR
NDOT6	Second derivative of mean motion	VARI, KALP, CARD, CARDINPUT, KAL, KALPAR
NM	Mean value of mean motion	VARI, KALP
NO	Mean motion at epoch	KAL, VARI, KALP, CARD, CARDINPUT, KALPAR
NODEM	Mean value of NODE	VARI, KALP
NODEO	Right ascension of ascending node	VARI, KALP, CARD, CARDINPUT, KAL, KALPAR
NODES	Short periodic term of NODE	VARI, KALP
OAZ	Azimuth offset for system	COMND, tracking routine, SEARCH
OEL	Elevation offset for system	COMND, tracking routine, SEARCH
OMEGL	Long periodic term for Omega	VARI, KALP
OMEGM	Mean periodic term for Omega	VARI, KALP
OMEGO	Original Omega	KALPAR, KAL, VARI, KALP, CARD, CARDINPUT
OSTIME	Sidereal time at the beginning of the day	PAR3, KALPAR, PAR5, KAL, TIME, tracking routines
OT	Time offset for system	COMND, tracking routines, TIME
OTSINCE	Time since epoch at beginning of the day	PAR3, KALPAR, PAR5, TIME, tracking routines
PK	Confidence matrix for the system	KAL, PKQX
QX	Error covariance matrix of system noise	KAL, PKQX
REVM	Mean number of revolutions from launch	VARI, KALP
REVO	Number of revolutions since launch to epoch	VARI, CARD, KALP, CARDINPUT, KAL, KALPAR

RMAG	Distance from center of earth to the satellite	VARI, KALP, ORBIT
RMGDT	First derivative of RMAG	VARI, KALP, ORBIT
ROWTK1	Observer-satellite vector in topocentric coordinates	KALP
RVDT	Transverse component of the velocity vector	VARI, KALP, ORBIT
RX, RY, RZ	ECI coordinates of the satellite	TRANS, VARI, KALP, ORBIT, PAR3, PAR5, KALPAR
SATNO	Satellite number	CARD, CARDINPUT, KAL-CARD, CARDINPUT
SINI	Sin (inclination)	VARI, KALP, ORBIT
SINLAT	Sin (latitude)	KAL, KALP, TRANS, PAR3, PAR5, KALPAR
SINS	Sin (sidereal time)	TRANS, ORBIT, TIME, PAR3, PAR5, KALPAR
SNODE	Sin (right ascension of the ascending node)	VARI, KALP, ORBIT
STIME	Sidereal time	KAL, ORBIT, TIME, PAR3 PAR5, KALPAR
TEMP	Temporary storage	VARI, KALP
TEMPL	Temporary storage	VARI, KALP
TIME	Time into search	SEARCH
TIME2	Time of last output to mount	MOUNT, main tracking routines
TRUEU	True anomaly	VARI, KALP, ORBIT
TSINCE	Time since epoch	VARI, TIME, KALP, COMND, KAL
VXY	Constants for radius of earth to the equator	PAR3, KAL, PAR5, KALPAR
VZ	Constants for radius of earth to pole	PAR5, KAL, PAR3, KALPAR
XBARK	Predicted state vector of orbital parameters	KALP
XHATK	Estimated state vector of orbital parameters	KAL
YR	Year of card parameters	CARD, COMND, CARDINPUT, KAL, KALPAR, KALP

SECTION 7

INTRODUCTION TO TV/NOVA INTERFACE

The Kalman filter technique for updating orbital parameters, described in Section 5 of this report, requires measurements of the actual position of the satellite in terms of its azimuth and elevation angles. A TV monitor on the mount console displays the same patch of sky at which the telescope is pointed. The required azimuth and elevation measurements can be obtained by means of adjustable cross hairs on the TV screen. This section describes the hardware designed and fabricated to obtain the adjustable cross hairs on the screen and to transfer the azimuth elevation angular measurements to the Active Tracking Program at operator command.

The camera interface positions two cross hairs on a video signal and relays the position of these cross hairs to the CPU with an accuracy equal to the resolution of the TV system.

To do this, the interface must be able to synchronize with the video signal and accurately maintain its position within the video signal. Referring to Figure 7-1, one can see the raster of a TV picture. We will call the solid raster, "frame one" of the picture, and dotted raster, "frame two." The two rasters are interlaced such that every other line of the scan is done by one frame. The slope of the scan is such that after one complete horizontal scan, the beam position has moved down two lines from the top.

The interlacing of the frames is done to provide as much detail in the picture as possible with a narrow bandwidth signal and also to avoid flicker on the screen.

This interlacing presents a problem to the interface. To achieve one line of resolution, the interface must draw half of a given line on one scan and the rest of the line on the next scan.

This is pointed out in that, after one half of a horizontal scan, the vertical position is one line below. To achieve some vertical height the line must now wait for one scan, start at one line above the end of the last line, and be drawn from the halfway point to the end.

In Figure 7-2, line #9 is the vertical line to be drawn. To do this, the first half of the fifth scan of the frame #2 raster is drawn. Then the second half of the fifth scan of the frame #2 is drawn. In Figure 7-3, the sixth line is to be drawn. To do this, the first half of the fourth scan of the second frame is drawn and the second half of the third scan of the first frame is drawn.

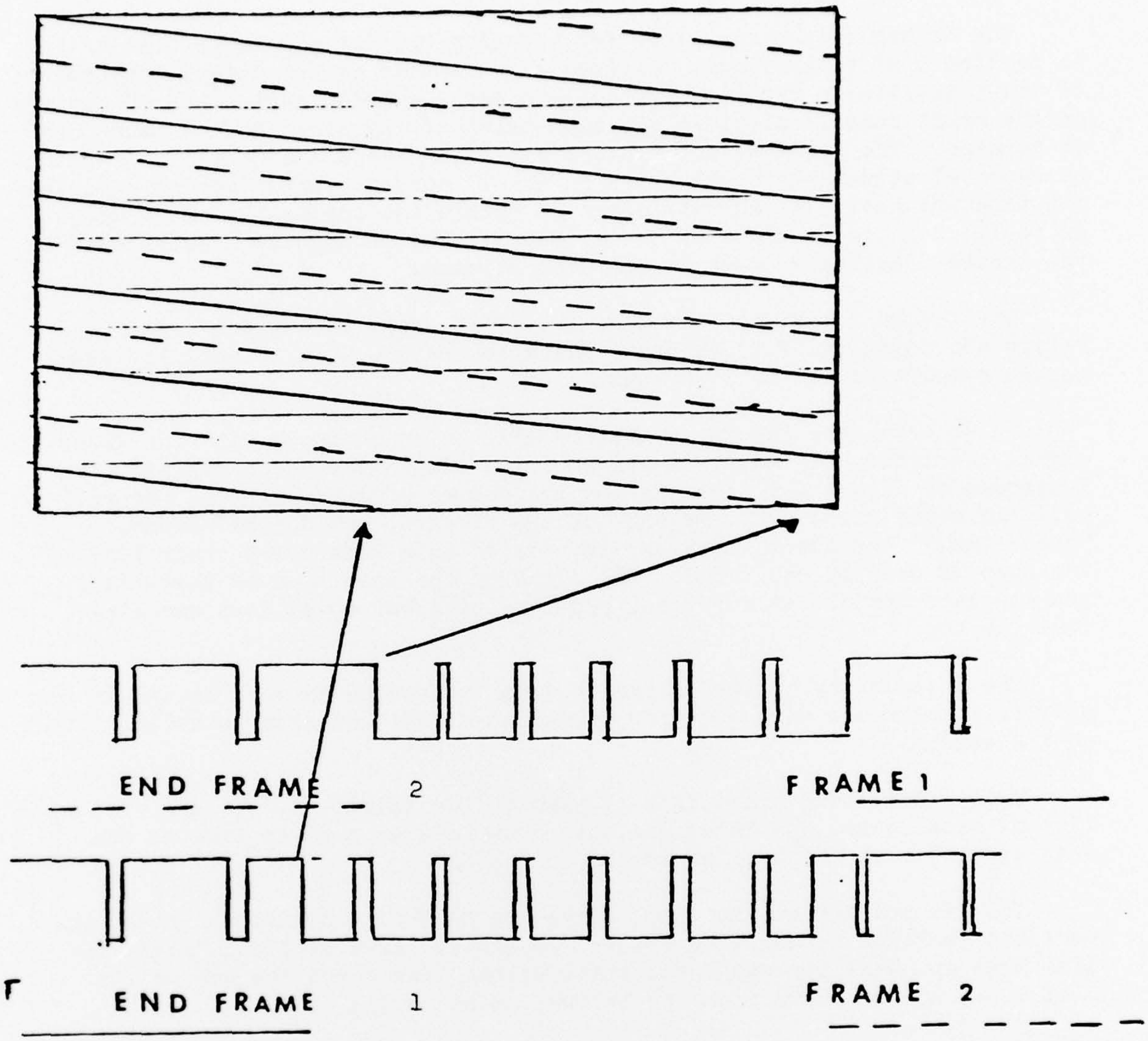


Figure 7-1 Raster Scan and Sync Pulse

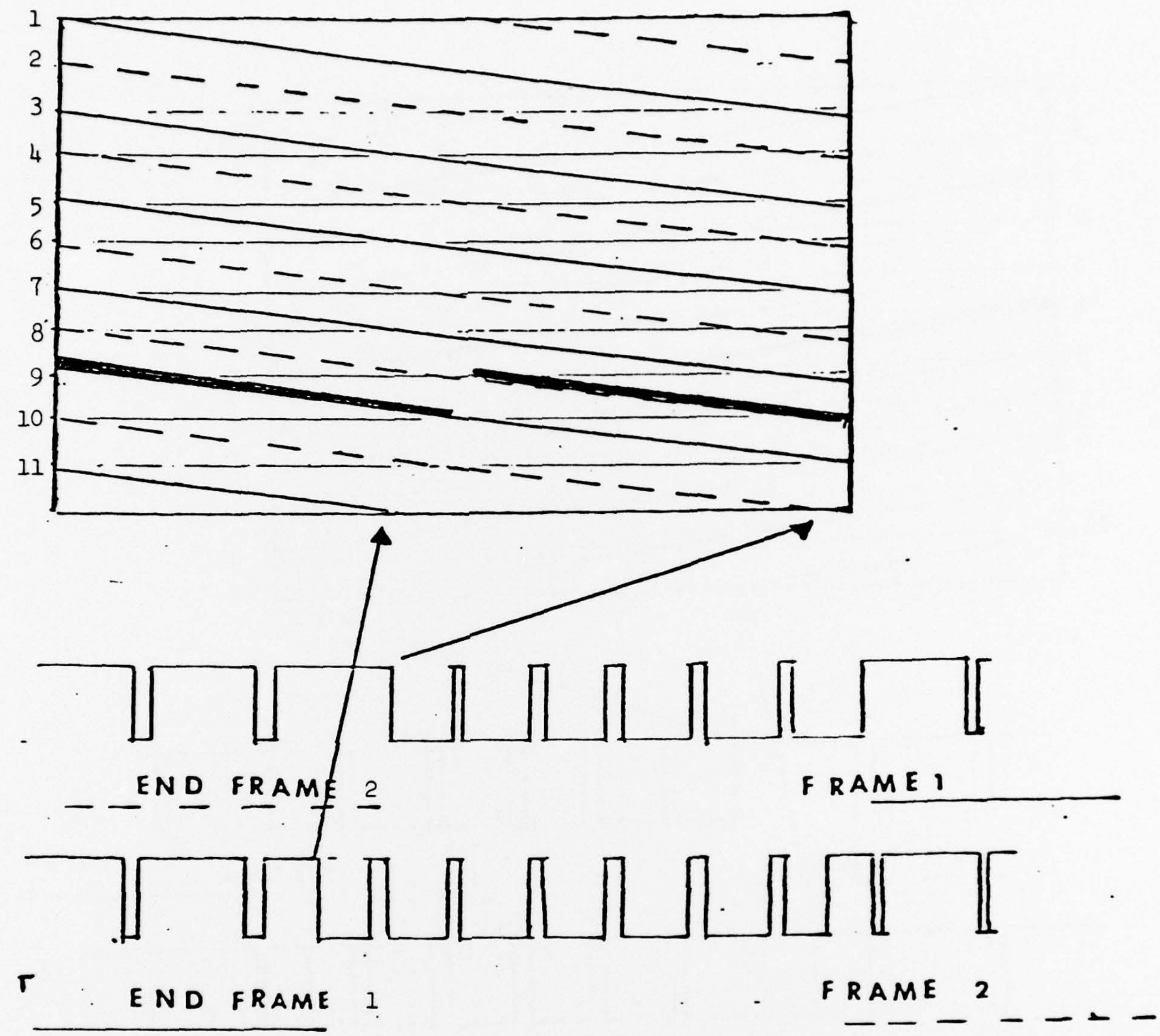


Figure 7-2 Raster Scan and Sync Pulse With Even Odd Horizontal Line

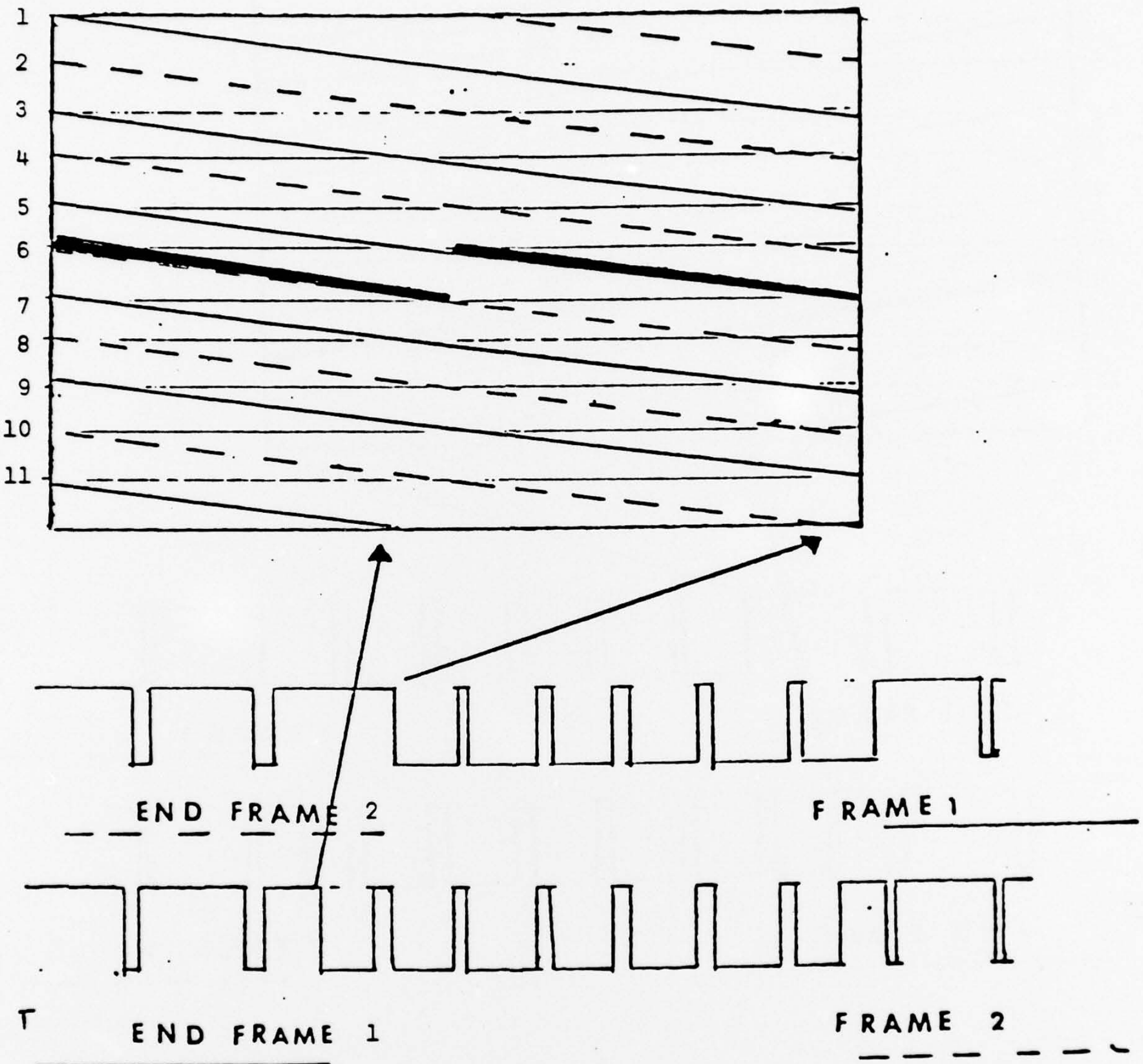


Figure 7-3 Raster Scan and Sync Pulse With Odd Even Horizontal Line

To accomplish this in hardware, each scan is divided into 525 half lines. Each half line is equal to one vertical increment. For each horizontal half line, the vertical counter is incremented one count. On the cross hairs can be summed as either a positive (black) cross hair or a negative (white) cross hair.

The digital position of the joystick is fed to the computer as azimuth and elevation angles with some scaling factor. The computer accepts the data after the operator pushes the transmit button.

There is only a simplified explanation of the circuitry. It is only intended to give the reader a general knowledge of the function of the interface.

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METRIC SYSTEM

BASE UNITS:

Quantity	Unit	SI Symbol	Formula
length	metre	m	...
mass	kilogram	kg	...
time	second	s	...
electric current	ampere	A	...
thermodynamic temperature	kelvin	K	...
amount of substance	mole	mol	...
luminous intensity	candela	cd	...

SUPPLEMENTARY UNITS:

plane angle	radian	rad	...
solid angle	steradian	sr	...

DERIVED UNITS:

Acceleration	metre per second squared	...	m/s
activity (of a radioactive source)	disintegration per second	...	(disintegration)/s
angular acceleration	radian per second squared	...	rad/s
angular velocity	radian per second	...	rad/s
area	square metre	...	m
density	kilogram per cubic metre	...	kg/m
electric capacitance	farad	F	A·s/V
electrical conductance	siemens	S	A/V
electric field strength	volt per metre	...	V/m
electric inductance	henry	H	V·s/A
electric potential difference	volt	V	W/A
electric resistance	ohm		V/A
electromotive force	volt	V	W/A
energy	joule	J	N·m
entropy	joule per kelvin	...	J/K
force	newton	N	kg·m/s
frequency	hertz	Hz	(cycle)/s
illuminance	lux	lx	lm/m
luminance	candela per square metre	...	cd/m
luminous flux	lumen	lm	cd·sr
magnetic field strength	ampere per metre	...	A/m
magnetic flux	weber	Wb	V·s
magnetic flux density	tesla	T	Wb/m
magnetomotive force	ampere	A	...
power	watt	W	J/s
pressure	pascal	Pa	N/m
quantity of electricity	coulomb	C	A·s
quantity of heat	joule	J	N·m
radiant intensity	watt per steradian	...	W/sr
specific heat	joule per kilogram-kelvin	...	J/kg·K
stress	pascal	Pa	N/m
thermal conductivity	watt per metre-kelvin	...	W/m·K
velocity	metre per second	...	m/s
viscosity, dynamic	pascal-second	...	Pa·s
viscosity, kinematic	square metre per second	...	m/s
voltage	volt	V	W/A
volume	cubic metre	...	m
wavenumber	reciprocal metre	...	(wave)/m
work	joule	J	N·m

SI PREFIXES:

Multiplication Factors	Prefix	SI Symbol
1 000 000 000 000 = 10^{12}	tera	T
1 000 000 000 = 10^{9}	giga	G
1 000 000 = 10^{6}	mega	M
1 000 = 10^{3}	kilo	k
100 = 10^{2}	hecto*	h
10 = 10^{1}	deka*	da
0.1 = 10^{-1}	deci*	d
0.01 = 10^{-2}	centi*	c
0.001 = 10^{-3}	milli	m
0.000 001 = 10^{-6}	micro	μ
0.000 000 001 = 10^{-9}	nano	n
0.000 000 000 001 = 10^{-12}	pico	p
0.000 000 000 000 001 = 10^{-15}	femto	f
0.000 000 000 000 000 001 = 10^{-18}	atto	a

* To be avoided where possible.

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